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# Dielectric and photoconductivity dependence study of four-wave mixing process in photorefractive materials

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**Abstract:** Phase-conjugate (PC) reflectivity is one of the most important parameters that characterize the four-wave mixing process in photorefractive (PR) materials. In this paper, the effect of the crystal thickness, modulation ratio and pump intensity ratio on the PC reflectivity of four-wave has been studied in case of the degenerate four-wave mixing process in PR materials. Also, the influence of photoconductivity and dielectric constant of PR materials on the PC reflectivity of the four-wave has been analyzed in case of the non-degenerate wave mixing process of PR materials. It has been found that the reflectivity of the PC wave for BGO and BSO have shown almost similar behavior like LiNbO<sub>3</sub> with the peak values observed at 1.21 pS/cm and 1.69 pS/cm. The present results showed that the reflectivity of the PC wave is different for all the materials of dielectric constant 32 (LiNbO<sub>3</sub>), 40 (BGO) and 56 (BSO) and is higher for higher value of dielectric constant, suggesting that the reflectivity of reflectivity of PC wave not only depends on the dielectric constant of the photorefractive materials but also strongly depends upon the photoconductivity of the materials. For lower value of coupling coefficient, it is observed at lower crystal thickness. The enhancement in the reflectivity of the optical phase-conjugate wave would greatly improve the performance of the devices based on the four wave mixing process. Such devices find applications in the areas like optical memories, information processing, real-time processing, beam steering, beam combining, resonators and pattern formation.

**Keywords:** Phase-conjugate four-wave mixing; Frequency detuning; Photoconductivity and dielectric constant of PR materials

# 1. Introduction

Miniaturization of electronic circuits is approaching a point where the electromagnetic interaction between the neighbouring elements affects their reliability. The optical waves are fast and do not interact with each other in linear media and hence, the optical based components are ideal candidates for expanding the frontier of computation. After the invention of the first laser by Maiman in 1960 this field of research, dealing with the soptical information processing (photonics) has grown considerably. It is, nowadays, one of the principal areas of development in science and technology [1–3]. Among the others, the major advantages of optics with respect to electronics are the intrinsic potential for the parallel computation and the much larger bandwidth of optical signals. For the construction of all-optical switching elements the use of nonlinear-optical materials is necessary. However, optical computing also depends on the availability of suitable nonlinear materials [2–8]. Requirements depend strongly on the kind of the targeted applications. Additional requirements are a controllable photosensitivity and light absorption of the materials. Due to its unique nature the photorefractive effect (photorefractive optics) can meet most of these requirements, by an appropriate material selection and a controlled doping treatment. The photorefractive effect is based on light

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induced charge transport in electro-optic materials [3]. Trapped charges are excited in a mobile state where they can freely move until they are re-trapped in an immobile state [2, 3]. In this process an internal space charge field is built up. Through the Pockels effect this spatially modulated field translates in a refractive index change, which is in turn detected by optical diffraction measurements. The advantages of photorefractive materials are low power consumption, strong beam coupling at low power levels, real time writing-erasure, storage possibility and parallel processing. Traditionally, experimentation in photorefractive optics leads, with theory lagging behind [1–18].

Nonlinear optical phase conjugation by degenerate fourwave mixing is an important technique with applications in many fields of science and technology [9-22]. In degenerate four-wave mixing, two counter propagating and intense light beams interact with a nonlinear medium, together with a less intense third one, a fourth beam is generated from the medium, which is the phase-conjugation of the third beam [20-39]. The main applications of degenerate four-wave mixing techniques are nonlinear spectroscopy, real-time holography, and phase conjugation [2–12, 33–41]. Optical phase conjugation is one of the most promising research fields of the nonlinear optics because of its potential applications to optical image processing, optical computing, adaptive optics, and optical interferometry and so on [1-19]. Phase conjugation by degenerate four-wave mixing has been demonstrated in numerous nonlinear media [4, 7, 8]. The photorefractive crystals, which can display strong nonlinear effects with milliwatt beams, are assumed to be the most promising media to generate phase conjugation. As a result, there have been much theoretical and experimental studies of phase conjugation in the photorefractive crystals [2-19]. It is generally believed that in some cases, when an external electric field [5-33] is applied to the crystal, the efficiency of the phase conjugation (or the phase conjugate reflectivity) is greatly enhanced [2–19]. However, the previous theoretical analyses were based on the assumption of the plane waves of the interacting beams. One may only get insight into the reflectivity of the phase conjugate beam [2-12].

To enhance the reflectivity of the phase conjugated (PC) wave, several methods have been proposed by several groups of researchers, e.g., (1) translating the four wavemixing (FWM) grating either by use of optical waves of slightly different frequencies or moving the crystal at a constant velocity [2–20] and (2) applying an electric field to the crystal [1–18, 21–29].

In the present paper we have described the reflectivity of the PC wave and analytical expression for the reflectivity has been derived analytically in nonlinear media (photorefractive media) assuming that the photorefractive (PR) medium is non-absorbing in case of degenerate and nondegenerate FWM [1–19]. The dependence of reflectivity in FWM on the pump intensity ratio has been considered earlier by several workers [9–12, 25–29]. We have introduced the concept of photoconductivity and dielectric constant of the PR medium in case of non-degenerate FWM process and studied its effect on PC reflectivity. For the enhancement in the reflectivity of the PC wave several groups of researchers have used an external electric field [1–6, 25–29]. In the case of degenerate four wave-mixing (DFWM), the influence of crystal thickness, modulation ratio and pump intensity ratio on the reflectivity of phaseconjugate wave in optical FWM has been studied in details.

### 2. Theoretical formulation

The optical FWM is a suitable method for the generation of the phase conjugated waves. In a FWM process an index grating (Fig. 1) is formed by the interference of four mixing beams. A nonlinear PR medium is pumped by two counter-propagating laser beams (2 and 3) and when a signal beam (1) is incident onto a PR crystal, a PC beam (4) is generated. This beam propagates opposite to the signal beam and is a time reversed replica of the signal beam. The interference pattern formed by the readout pump beam 3 and the PC beam has the same spatial periodicity and orientation as the pattern formed by the signal beam 1 and the writing pump beam 2 (this is simply due to the perfect phase- matching condition in DFWM). However, the strength of the PC signal depends strongly on the phase shift between the interference pattern of the beams 3 and 4 and the overall index grating generated by the FWM process [1-12, 22-36]. Considering the interaction of four laser beams having the same frequency  $\omega$  in a PR medium, the resultant electric field due to the four beams may be written as,

$$E = \sum_{j=1}^{4} E_j \exp\left[i\left(\omega t - \vec{k}_j \cdot \vec{r}\right)\right]$$
(1)

where  $E_1, E_2, E_3, E_4$  are the complex amplitudes and



Fig. 1 Schematic diagram of four wave mixing

 $\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4$  are the electric wave vectors of the four beams 1, 2, 3 and 4, respectively. In Eq. (1), all the four wave electric vectors are taken to be coplanar and all the waves to be normally polarized to the plane. If  $E_2 = E_4 = 0$ , there are only two waves and only one PR grating. The coupled equations for the electric fields  $E_1$  and  $E_2$  of the beams 1 and 2 can be written as [2, 3, 16–19]

$$\frac{\mathrm{d}E_1}{\mathrm{d}z} = -\frac{\Gamma}{2I_0} \left( E_1 E_2^* \right) E_2 \tag{2}$$

$$\frac{dE_2}{dz} = -\frac{\Gamma^*}{2I_0} \left( E_1^* E_2 \right) E_1 \tag{3}$$

where one assumes,  $\alpha = 0$  (i.e., non-absorbing media) and considering the two-wave mixing in PR materials with extraordinary polarized incident beams, the complex coupling constant  $\Gamma$  is given by,

$$\Gamma = \frac{i \pi \Delta n \exp(-i \Phi)}{\lambda \cos(\theta)} \tag{4}$$

where  $\lambda$  is the wavelength of the laser beam;  $\theta$  is the half the angle between the incident beams 1 and 2 inside the photorefractive medium;  $i = \sqrt{-1}$ ;  $\Delta n$  is the saturation value of the photo-induced index change in the photorefractive materials and  $\Phi$  is the phase shift between the interference pattern and volume index grating. In diffusion regime, without external electric field applied to the crystal,  $\Phi = \frac{\pi}{2}$ . The explicit expression for  $\Delta n$  [2–19, 25–41] is given by,

$$\Delta n = \left(\frac{r_e n^3 E_q m}{2}\right) \left[\frac{E_D^2}{\left(E_D + E_q\right)^2}\right] \tag{5}$$

where  $r_e$  is the electro-optic coefficient of the PR crystal,  $m = \frac{2\sqrt{I_1I_2}}{(I_1 + I_2)}$  is the average modulation ratio of the light pattern; n is the bulk refractive index at the laser wavelength;  $E_D = \frac{K k_B T}{e}$  is the diffusion field  $K = 2\pi/\Lambda$  is the magnitude of the grating wave vector  $\vec{K}$ ;  $\Lambda = \frac{\lambda}{2 \sin \theta}$  is the grating period;  $k_B$  is the Boltzmann constant; T is the absolute temperature; e is the magnitude of the electronic charge;  $E_q = \frac{eN_A}{\epsilon_r \epsilon_0 K}$  is the saturation space charge field;  $N_A$  is the acceptor density;  $\epsilon_0$  is the vacuum permittivity and  $\epsilon_r = \left(\frac{\epsilon}{\epsilon_0}\right)$  is the relative material permittivity (dielectric constant), and  $\epsilon$  is the material permittivity [2–12, 22–33].

The term  $E_1E_2^*$  in Eqs. (2) and (3), represents the amplitude of the grating formed by the waves 1 and 2. Since in case of the four-wave mixing process for the phase conjugation, the wave vectors come in two oppositely directed pairs, one can write,

$$\vec{k}_2 = -\vec{k}_3$$
 and  $\vec{k}_4 = -\vec{k}_1$  (6)

Further, we assume that if these waves enter into the medium symmetrically as shown in Fig. 1, four different gratings are formed, which are represented by  $(E_1E_2^* + E_3E_4^*)$ ,  $(E_2E_4^* + E_1E_3^*)$ ,  $(E_2E_3^*)$  and  $(E_1E_4^*)$ . The first one of the above is a transmission grating; while the second one is a reflection grating. The remaining two gratings are known as the 2k gratings (as these have wave vector  $2\vec{k}$ ). Here, it is assumed that only the transmission grating  $(E_1E_2^* + E_3E_4^*)$  gives rise to strong interaction [1-15, 22-41].

Practically, the predominance of one grating is common due to the dependence of the space charge field (SCF) on the grating periods, grating orientations and coherence between the beams [4–13, 25–29]. These conditions lead to the following coupled equations for the amplitudes  $E_1, E_2, E_3$  and  $E_4$ :

$$\frac{\mathrm{d}E_1}{\mathrm{d}z} = -\frac{\Gamma}{2I_0} \left( E_1 E_2^* + E_3 E_4^* \right) E_2 \tag{7}$$

$$\frac{\mathrm{d}E_2}{\mathrm{d}z} = \frac{\Gamma^*}{2I_0} \left( E_1^* E_2 + E_3^* E_4 \right) E_1 \tag{8}$$

$$\frac{dE_3}{dz} = \frac{\Gamma}{2I_0} \left( E_1 E_2^* + E_3 E_4^* \right) E_4 \tag{9}$$

$$\frac{\mathrm{d}E_4}{\mathrm{d}z} = -\frac{\Gamma^*}{2I_0} \left( E_1^* E_2 + E_3^* E_4 \right) E_3 \tag{10}$$

where  $I_0 = |E_1|^2 + |E_2|^2 + |E_3|^2 + |E_4|^2 = I_1 + I_2 + I_3 + I_4$ , is the averaged total intensity of all the four beams [1–7].

In arriving at the above set of nonlinear coupled wave equations, the material absorption is ignored. There are two contributions to the grating, one due to the beams 1 and 2, the other due to the beams 3 and 4. These two contributions are identical in grating periods and orientations and have the same coupling coefficient  $\Gamma$  [2–4].

In optical phase conjugation using FWM, the beams 2 and 3 are designated as the pump beams and the beam 1 as the signal beam. In the undepleted-pump approximation, one assumes the condition,

$$|\mathbf{E}_1|^2, |\mathbf{E}_4|^2 \ll |\mathbf{E}_2|^2, |\mathbf{E}_3|^2 \tag{11}$$

so that  $E_2$  and  $E_3$  may be taken as constant during the propagation through the crystal, i.e.,  $\frac{dE_2}{d_z} = \frac{dE_3}{d_z} = 0$  (12).

Ten, the coupled Eqs. (7) and (10) become,

$$\frac{\mathrm{d}E_1}{\mathrm{d}z} = \frac{\Gamma}{2I_0} |E_2|^2 E_1 - \frac{\Gamma}{2I_0} E_2 E_3 E_4^* \tag{13}$$

$$\frac{\mathrm{d}E_4^*}{\mathrm{d}z} = \frac{\Gamma}{2I_0} |E_3|^2 E_4^* - \frac{\Gamma}{2I_0} E_2^* E_3^* E_1 \tag{14}$$

where  $|E_2|^2 = E_2 E_2^*$  and  $|E_3|^2 = E_3 E_3^*$ . Under the undepleted

pump approximation Eqs. (13) and (14) can be integrated. The solutions, of Eqs. (13) and (14) with the boundary conditions  $E_4(L) = 0$  and  $E_1(0)$  are respectively given by,

$$E_1 = \frac{\exp\left(-\frac{1}{2}\Gamma z\right) + q\exp\left(-\frac{1}{2}\Gamma L\right)}{1 + q\exp\left(-\frac{1}{2}\Gamma L\right)}E_1(0) \tag{15}$$

$$E_4^*(z) = \left(\frac{E_3^*}{E_2}\right) \left[\frac{\exp\left(-\frac{1}{2}\Gamma z\right) - \exp\left(-\frac{1}{2}\Gamma L\right)}{1 + q\exp\left(-\frac{1}{2}\Gamma L\right)}\right] E_1(0) \quad (16)$$

where q is the pump intensity ratio given by,

$$q = \frac{|E_3|^2}{|E_2|^2} = \frac{I_3}{I_2}$$
(17)

Thus, it is evident that for the PR medium with real and positive  $\Gamma$ , both the amplitudes  $E_1(z)$  and  $E_4^*(z)$  are decaying exponentially in the medium. The beam 4 is generated by the FWM process; starts with  $E_4 = 0$  at z = L and grows exponentially along the z-direction.

According to Eq. (16), the amplitude  $E_4$  of the beam 4 at z = 0 is proportional to  $E_1^*(0)$ . Hence, one could say that beam 4 is the PC of the signal beam 1. The reflection coefficient of the PC beam is given by,

$$\rho = \frac{E_4(0)}{E_1^*} = \left(\frac{E_3}{E_2^*}\right) \left[\frac{1 - \exp\left(-\frac{1}{2}\Gamma^*L\right)}{1 + q\exp\left(-\frac{1}{2}\Gamma^*L\right)}\right]$$
(18)

Here,  $\rho$  is a complex number with a phase which depends on  $E_2$ ,  $E_3$  and  $\Gamma$ . In PR media with strong coupling coefficient, the term  $\exp(-\frac{1}{2}\Gamma * L)$  is either very large or very small and therefore, the phase of  $\rho$  is the same as that of  $E_2 E_3$ . The PC beam carries the phase information of the pump beams.

Eq. (18) leads to the expression for the PC reflectivity  $R_{f}$ ,

$$R_f = |\rho|^2 = \left| \frac{\sin h(\frac{1}{4}\Gamma L)}{\cos h(\frac{1}{4}\Gamma L - \ln \sqrt{q})} \right|^2$$
(19)

Substituting the value of  $\Gamma$  from Eq. (9) and using Eq. (10) the phase conjugate reflectivity assumes the following form,

$$R_{f} = \left| \frac{\sin h^{2} \left\{ \left( \frac{1}{8} \frac{\pi n^{3} r_{e} m}{\lambda \cos \theta} \right) \left( \frac{E_{D}}{1 + \frac{E_{D}}{E_{q}}} \right) L \right\}}{\cos h^{2} \left\{ \left( \frac{1}{8} \frac{\pi n^{3} r_{e} m}{\lambda \cos \theta} \right) \left( \frac{E_{D}}{1 + \frac{E_{D}}{E_{q}}} \right) L - \ln \sqrt{q} \right\}} \right|$$
(20)

Eq. (20) shows that the PC reflectivity is an increasing function of m and L. For the symmetric pumping case, i.e., q = 1, Eq. (20) becomes,

$$R_f = \tanh^2 \left\{ \frac{1}{8} \frac{\pi n^3 r_e m}{\lambda \cos \theta} \left( \frac{E_D}{1 + E_D / E_q} \right) L \right\}$$
(21)

The value of  $R_f$  given by Eq. (3.21) is always less than

unity. To get higher reflectivity one has to have asymmetric pumping. The reflectivity acquires the maximum value for  $q = \exp(\frac{1}{2}\Gamma L)$  and is given by,

$$R_f = \sinh^2 \left\{ \frac{1}{8} \frac{\pi n^3 r_e m}{\lambda \cos \theta} \left( \frac{E_D}{1 + E_D / E_q} \right) L \right\}$$
(22)

So far we have assumed that all the waves have exactly the same frequency. We now consider the case when the frequencies of all the four waves are different. Let the resultant electric field be written as,

$$\vec{E} = \sum_{j=1}^{4} \sum_{j} \exp\left[i\left(\omega_{j} t - \vec{k_{j}} \cdot \vec{r}\right)\right]$$
(23)

where  $\omega_1$ ,  $\omega_2$ ,  $\omega_3$ , and  $\omega_4$  are the frequencies of the waves. The grating approximation used in case of the DFWM process is valid with the wave vectors and frequencies satisfying the conditions,

$$\vec{k_2} - \vec{k_1} = \vec{k_4} - \vec{k_3} = \vec{k}$$
(24)

$$\omega_2 - \omega_1 = \omega_4 - \omega_3 = \Omega \tag{25}$$

where  $\overline{k}$  is the wave vector of the grating and  $\Omega$  is the angular frequency of the grating. For the four amplitudes the coupled differential equations are Eqs. (7)–(10). The coupling coefficient  $\Gamma$  for the non-degenerate case is given by,

$$\Gamma = \frac{\gamma_0}{1 + i\Omega\tau} = \frac{\gamma_0}{1 + \Omega^2\tau^2} - \frac{i\gamma_0\Omega\tau}{1 + \Omega^2\tau^2}$$

$$\Gamma = \gamma + i\beta$$
(26)

where  $\gamma_0$  is the coupling coefficient for the case of degenerate two wave mixing [5–16],  $\tau$  is the response time of the PR medium and is given by the relation [3–20–29],

$$\tau = \left(\frac{\varepsilon_r \, \varepsilon_0}{\sigma_p}\right) \tag{27}$$

where  $\varepsilon_r = \left(\frac{\varepsilon}{\varepsilon_0}\right)$  is the dielectric constant [1–6, 25–29],  $\varepsilon$  is the material permittivity,  $\varepsilon_0$  is the permittivity of free space, and  $\sigma_p$  is the photoconductivity of the photorefractive material, respectively [1–7]. In arriving at Eq. (27) we have assumed that the dark conductivity is negligibly small in comparison with the photoconductivity. Substituting the value of  $\tau$  in Eq. (26) we have,

$$\Gamma = \frac{\gamma_0}{1 + \Omega^2 \tau^2} - \frac{i\gamma_0 \Omega \tau}{1 + \Omega^2 \tau^2} = \frac{\gamma_0 \sigma_p^2}{\sigma_p^2 + \Omega^2 (\varepsilon_r \varepsilon_0)^2} - \frac{i\gamma_0 \sigma_p \varepsilon_r \varepsilon_0}{(\sigma_p^2 + \Omega^2 (\varepsilon_r \varepsilon_0)^2}$$
(28)

Using Eqs. (19) and (28) the PC reflectivity  $R_f$  becomes,

$$R_{f} = \left| \frac{\sin h^{2} \left[ \frac{1}{8} \left\{ \frac{\gamma_{0} \sigma_{p}^{2}}{\sigma_{p}^{2} + \Omega^{2}(\varepsilon_{r}\varepsilon_{0})^{2}} - \frac{i\gamma_{0} \sigma_{p}^{2}\varepsilon_{r}\varepsilon_{0}}{\sigma_{p}^{2} + \Omega^{2}(\varepsilon_{r}\varepsilon_{0})^{2}} \right\} L \right]}{\cos h^{2} \left[ \frac{1}{8} \left\{ \frac{\gamma_{0} \sigma_{p}^{2}}{\sigma_{p}^{2} + \Omega^{2}(\varepsilon_{r}\varepsilon_{0})^{2}} - \frac{i\gamma_{0} \sigma_{p}^{2}\varepsilon_{r}\varepsilon_{0}}{\sigma_{p}^{2} + \Omega^{2}(\varepsilon_{r}\varepsilon_{0})^{2}} \right\} L - \ln \sqrt{q} \right]} \right|$$
(29)

From the above Eq. (29) it could be seen that the PC reflectivity is an increasing function of L. It is not only a function of L but also a function of photoconductivity, dielectric constant of the materials and oscillation frequency shift and pump intensity ratio of the interfering beams [1–15, 25–29].

#### 3. Results and discussion

From Eq. (20) it is clear that the PC reflectivity  $R_f$  depends on the modulation ratio (*m*), crystal thickness (*L*) and pump intensity ratio (*q*). For the calculation of the PC reflectivity of PC wave the following parameters have been selected  $\theta = 13.8^{\circ}$ ;  $k_B = 1.38 \times 10^{-23}$ ; T = 300 K;  $N_A = 0.95 \times$  $10^{22}$  m<sup>-3</sup>; n = 2.54;  $\varepsilon_0 = 8.85 \times 10^{-12}$  C<sup>2</sup>/Nm<sup>2</sup>; material permittivity  $\varepsilon = 56\varepsilon_0$ ;  $r_e = 8 \times 10^{-12}$  m/v;  $\lambda = 532 \times$  $10^{-9}$  m;  $E_D = [Kk_BT/e]$ ,  $E_q = \begin{bmatrix} eN_A / \varepsilon_0 \varepsilon K \end{bmatrix}$  and  $e = 1.6 \times$  $10^{-19}C$  [14, 25–29].

Variation of PC reflectivity  $R_f$  with the pump intensity ratio q for constant value of L(= 2 cm) for various values of modulation ratio m(= 1, 10, 100) and for constant value of m(= 1) for different values of crystal thickness L(= 2, 3,4 cm) are shown in Fig. 2(a) and (b), respectively. From Fig. 2(a) it is obvious that for the lower value of m (= 1), the phase conjugate reflectivity in the FWM increases with the increasing pump intensity ratio (q) and reaches its saturation value after a certain value of q but for higher value of m (> 10), the reflectivity of the PC wave shows linear behavior with the pump intensity ratio and the coincidence behavior of reflectivity of the PC beam for m(= 10) and for m(= 100) indicates that it is unaffected for higher values of m (> 10). From Fig. 2(b), one could see that for thinner crystals ( $L \le 2$  cm), the PC reflectivity in the FWM shows behavior similar to that in Fig. 2(a) but for thicker crystals ( $L \ge 3$  cm), the reflectivity of the PC wave increases with the increasing crystal thickness of the photorefractive materials. Thus, one could conclude that for a given value of the pump intensity ratio the PC reflectivity in the FWM process could be greatly enhanced by selecting thicker crystals ( $L \ge 3$  cm) of PR materials of with higher value of modulation ratio (m > 10).

Figure 3(a) and (b) shows variation of the reflectivity of the PC wave in the FWM as a function of crystal thickness of the material for constant value of q = 1000 at various values of m(=1, 3, 5) and for constant value of m(=1) at different values of pump intensity ratio q(=100, 500,1000), respectively. From these figures, it is evident that the reflectivity of the PC wave increases with the increasing crystal thickness and reaches its saturation value after a certain value of L and magnitude of the saturation value of reflectivity is the same for all the values of m. The saturation is achieved with thinner crystals for high m values Fig. 3(a). Thus, for a fixed value of q, higher reflectivity of the PC beam could be obtained at a much lower thickness of the PR material by increasing the modulation ratio. From Fig. 3(b), it is evident that the PC reflectivity in the FWM increases with the crystal thickness and reaches its saturation value after a certain value of L. The magnitude of the saturation value of reflectivity increases with the increasing value of the pump intensity ratio. From Fig. 3(a) and (b) one may conclude that for a given value of the crystal thickness (L), the reflectivity of the PC beam



Fig. 2 (a) Variation of PC reflectivity  $R_f$  with the pump intensity ratio q for L = 2 cm and various values of modulation ratio m(= 1, 10, 100). (b) Variation of PC reflectivity  $R_f$  with the pump intensity ratio q for m = 1 and different values of crystal thickness L(= 2, 3, 4 cm)



**Fig. 3** (a) Variation of the reflectivity of the PC wave with crystal thickness of the material for q = 1000 at various values of m(= 1, 3, 5). (b) Variation of the reflectivity of the PC wave with crystal

in the FWM is found to be higher (1000%) for higher value of the pump intensity ratio and lower value of modulation ratio (*m*) provided the crystal thickness is not less than 1.5 cm which is better than earlier reported value 670% [20–29].

Variation of PC reflectivity  $R_f$  with the crystal thickness L for constant value of  $\gamma_0 = 10 \text{ cm}^{-1}$ ,  $\varepsilon = 56$ ,  $\omega = 0.01 \text{ Hz}$ and  $\sigma_p = 2 \text{ pS/cm}$  for various values of input intensity ratio q(= 10, 50, 100) and for constant value of  $\gamma_0 = 10 \text{ cm}^{-1}$ ,  $\varepsilon = 56$ ,  $\omega = 0.001 \text{ Hz}$  and  $\sigma_p = 2 \text{ pS/cm}$  for different values of input intensity ratio q(= 10, 50, 100) are shown in Fig. 4(a) and (b), respectively. The reflectivity of PC wave increases with the crystal thickness and reaches its optimum value at certain value of crystal thickness and then decreases and acquiring its lowest value. The peak intensity of PC reflectivity is different for different values of input intensity ratio (Fig. 4b) and for higher value of input



thickness of the material for m = 1 at different values of pump intensity ratio q(= 100, 500, 1000)

intensity ratio the peak intensity of reflectivity PC wave is found at higher crystal thickness and vice versa. For lower values of frequency detuning the peak intensity of reflectivity of PC wave shifted toward lower crystal thickness side.

Figure 5(a), (b), and (c) shows the variation of PC reflectivity  $R_f$  with the crystal thickness *L* for constant value of q = 10,  $\varepsilon = 56$ ,  $\omega = 0.01$  Hz and  $\sigma_p = 2$  pS/cm for various values of input intensity ratio  $\gamma_0 = (5, 10, \text{ and } 15 \text{ cm}^{-1})$ , for constant value of q = 50,  $\varepsilon = 56$ ,  $\omega = 0.01$  Hz and  $\sigma_p = 2$  pS/cm for various values of input intensity ratio  $\gamma_0 = (5, 10, \text{ and } 15 \text{ cm}^{-1})$  and for constant value of q = 100,  $\varepsilon = 56$ ,  $\omega = 0.01$  Hz and  $\sigma_p = 2$  pS/cm for various values of input intensity ratio  $\gamma_0 = (5, 10, \text{ and } 15 \text{ cm}^{-1})$  and for constant value of q = 100,  $\varepsilon = 56$ ,  $\omega = 0.01$  Hz and  $\sigma_p = 2$  pS/cm for various values of input intensity ratio  $\gamma_0 = (5, 10, \text{ and } 15 \text{ cm}^{-1})$ , respectively. The reflectivity of the PC wave increases with the crystal thickness and the increment is very low up to 0.4 mm but a drastic change is noticed in



Fig. 4 (a) Variation of phase conjugate reflectivity with the crystal thickness of the materials for different input intensity ratio. (b) Variation of phase conjugate reflectivity with the crystal thickness of the materials for different input intensity ratio



**Fig. 5** (a) Variation of phase conjugate reflectivity with the crystal thickness of the materials for different coupling coefficient. (b) Variation of phase conjugate reflectivity with the crystal thickness of the

the range 0.4–0.6 mm for  $\gamma_0 = 15$  and is different for different values of coupling coefficient. For lower value of coupling coefficient it is observed that the peak intensity of reflectivity of PC wave occur at higher thickness and for higher values of coupling coefficient it is observed at lower crystal thickness. If we select the pump intensity ratio is higher the peak intensity of reflectivity of the PC wave is shifted toward the higher crystal thickness side.

Variation of PC reflectivity  $R_f$  with the photoconductivity of the photorefractive materials for constant value of crystal thickness L = 2 cm, coupling coefficient = 10 cm<sup>-1</sup>, input intensity ratio q = 100 and frequency detuning = 0.01 Hz for various photorefractive materials (LiNbO<sub>3</sub>, BGO and BSO) is shown in Fig. 6. The reflectivity of the PC wave for LiNbO<sub>3</sub> is extremely low up to 0.80 pS/cm but a drastic increase is noticed in the range 0.85–1.10 pS/cm with a peak at 0.97 pS/cm which is of the order of 10<sup>5</sup>. Above 1.10 pS/cm it decreases acquiring extremely low value again. The reflectivity of the PC wave

materials for different coupling coefficient. (c) Variation of phase conjugate reflectivity with the crystal thickness of the materials for different coupling coefficient



Fig. 6 Variation of phase conjugate reflectivity with the photoconductivity of the materials for different photorefractive materials

for BGO and BSO have shown almost similar behavior like LiNbO<sub>3</sub> and peak value observed at 1.21 pS/cm and 1.69 pS/cm, respectively. The reflectivity of the PC wave is different for all the materials of dielectric constant 32 (LiNbO<sub>3</sub>), 40 (BGO) and 56 (BSO) and is higher for higher value of dielectric constant. Thus, one could conclude that the reflectivity of the PC wave not only depends on the dielectric constant of the photorefractive materials but also strongly depends upon the photoconductivity of the materials.

# 4. Conclusions

Photoconductivity and dielectric constant dependence reflectivity of the phase conjugated wave in four wave mixing process of photorefractive materials has been studied in case of non-degenerate and degenerate wave mixing. It is found that in case of DFWM process in PR materials the reflectivity of the PC wave increases with the crystal thickness and reaches its saturation value after a certain value of L and magnitude of the saturation value of reflectivity is the same for all values of m. The saturation is achieved with thinner crystals for higher m values. It is found that for a given value of the crystal thickness (L> 1.5 cm), the reflectivity of the PC beam in the FWM is higher for higher value of pump intensity ratio and lower value of modulation ratio (m). The peak intensity of PC reflectivity is different for different values of input intensity ratio and for higher value of input intensity ratio the peak intensity of reflectivity PC wave is found at higher crystal thickness. The reflectivity of the PC wave is not only depending on the dielectric constant of the photorefractive materials but also it strongly depends upon the photoconductivity of the materials. However, for the application point of view, this enhancement in reflectivity of optical phase-conjugate wave could be used in many fields of science and technology [1-6].

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