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Frequency detuning dependence of oscillation condition for a semilinear photorefractive optical resonator in a photorefractive material with reflection gratings

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ABSTRACT

Frequency detuning dependence of four-beam coupling in a photorefractive crystal pumped with two counter-propagating waves for a semilinear coherent optical resonator on the oscillation conditions has been analyzed in the case of non-degenerate-wave mixing under the slowly varying amplitude approximation method. Self oscillation can be achieved when the gain arising from the four-beam coupling is large enough to overcome the cavity loss. The effects of frequency detuning (i.e., non-degeneracy), dielectric constant and photoconductivity of the photorefractive materials on the performance of the semilinear photorefractive coherent resonator with the reflection grating configuration have also been studied in detail. The phase-conjugate reflectivity of the pumped crystal and oscillation intensity has been calculated for different input pump beam intensity ratio, intensity reflectivity of the conventional mirrors, degenerate energy coupling strength of the interacting beams. It has been found that for the higher value of the photoconductivity σ_p(>2.0 pS/cm) of photorefractive crystal, the semilinear resonator can oscillate at almost any frequency detuning (Ω) of the oscillation beam with respect to the fixed frequency of the pump waves whereas for the lower value of photoconductivity σ_p(<0.1 pS/cm) oscillation occurs only when the frequency detuning is limited to small region around $\Omega = 0$. But reverse of the case is found for dielectric constant (ε), pump intensity ratio (p) and conventional mirror reflectivity (R).

1. Introduction

The semilinear photorefractive optical resonator with two counter-propagating pump waves [1] was studied recently in detail both theoretically and experimentally with a photorefractive crystal as the gain medium [2–12]. Its cavity is formed by an ordinary feedback mirror and a photorefractive crystal that serves as an amplifying phase-conjugate mirror pumped by two counter-propagating coherent light waves of the same frequency as result of the selective amplification of the noisy photorefractive grating via four-wave mixing processes. The efficiency of coherent oscillation depends on the particular geometry of the wave-mixing and on material parameters of the crystal. Optimization and deliberate tuning of the performance of photorefractive oscillators are important for their potential applications in lasers with capability for intra-cavity phase-distortion compensation [13,14], optical image transmission, computing systems, self-adjustable interferometers, logical and bistable elements, and optical non-linear associative memory [15]. It served also as a model system for investigation of deterministic chaos [16,2].

The single-frequency oscillation in an externally pumped phase-conjugate resonator (i.e., semilinear photorefractive oscillator) [17,18] becomes unstable above a certain critical value of the coupling strength [5]. With the removed degeneracy in frequency the initially static index grating starts to move and these results in an additional non-linear phase shift of the phase conjugate wave inside the cavity. After two consecutive round trips the nonlinear phase of the oscillation wave does not vanish; moreover it is accumulated with the successive number of round trips in the cavity. To ensure oscillation, this phase shift should either be compensated by any possible means or it should become equal to 2π to restore the in-phase addition of the partial components of the oscillation wave after each double round trip of the cavity [19]. One way to compensate for an undesirable phase shift is to use a frequency shifted feedback [20]. Another way consists of a slight misalignment (of the order of a fraction of milliradian) of the two pump waves [8]. The enhancement of the phase conjugate reflectivity by frequency shift in the case of a perfectly aligned oscillator is independent on the sign of the shift. By the same way, the enhancement of the phase conjugate reflectivity by pump misalignment in the case of a non-frequency shifted oscillator is independent on the sign of this tilt.

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Nevertheless if the both effects are considered simultaneously, this symmetry is broken. For misaligned pump waves the calculated beat frequency in the oscillation spectrum for a negative detuning is identical to that for a positive detuning of the same amplitude [8].

The onset of coherent oscillation in this geometry, as it has been shown recently, is similar to the second-order phase transition [3,4]. Exactly at the coherent oscillation threshold only one photorefractive grating with a well defined grating vector emerges from the multitude of low-amplitude arbitrarily oriented noisy gratings responsible for light-induced (nonlinear) scattering in photorefractive crystals. The amplitude of this selected grating increases rapidly with the increasing coupling strength which is a control parameter here. The light wave diffracted from the developing grating is the oscillation wave. Its amplitude normalized to the amplitude of the pump wave is an order parameter [21,22]. Predominating attention has been paid to resonators with transition grating. This is a consequence of space charge limitations that due to insufficient effective trap density of the photorefractive samples used. A rather high Debye screening length in comparison with small grating spacing of the reflection grating leads to considerably smaller gain factors than that calculated from electro-optic properties and the diffusion field only [15]. If, however, a photorefractive sample with a sufficiently large trap density is found, the advantages of the reflection grating can be fully used, since the ultimate coupling coefficient is increased compared to the transmission geometry.

We are interested in this paper in the dynamics of the semilinear photorefractive coherent resonator with a non-degenerate four wave-mixing. In this work, we have been analytically solved the problem of frequency detuning dependence of four-beam coupling in photorefractive crystal pumped with two counterpropagating pump beams for a semilinear photorefractive coherent optical resonator with dominating reflection gratings in the case of non-degenerate wave mixing. The phase-conjugate reflectivity, oscillation conditions, threshold conditions and the intensity of oscillation are studied for a semilinear coherent optical resonator under the appropriate boundary conditions. In the earlier published literatures [7–10] the effects of the frequency detuning (i.e., non-degeneracy), energy coupling strength of the interacting beams, intensity reflectivity of the conventional mirrors and input pump beam intensity ratio on the intensity of oscillation and phase-conjugate reflectivity of the pumped crystals have not been explored in detail. Moreover, the effects of the dielectric constant and photoconductivity of the photorefractive materials on the intensity of oscillation and the phase-conjugate reflectivity of the pumped crystals have not been considered earlier. In this paper the above effects are considered in details and discussed on its performances.

2. Theoretical description

2.1. Semilinear photorefractive coherent resonator

The schematic representation of the coherent optical resonator under consideration is shown in Fig. 1.

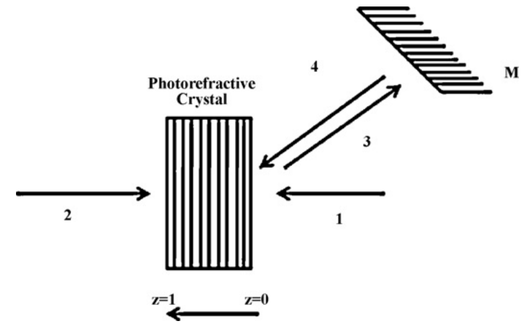

Fig. 1. Semilinear photorefractive optical resonator.

It is based on four-wave mixing in a photorefractive crystal pumped with two counterpropagating waves, 1 and 2, enter the photorefractive sample (photorefractive crystal) from the opposite faces. Any noisy wave propagating in direction 4 (part of the pump radiation scattered from optical imperfections of the sample) give rise to phase-conjugate wave 3, i.e., the photorefractive crystal with the length l serves as a phase conjugate mirror and forms together with the conventional mirror M a semilinear photorefractive optical oscillator. This conventional mirror M reflects the light scattered from the sample back to the pumped area of the sample and the conventional mirror M with intensity reflectance R serves as the second cavity mirror. The ratio of intensities of the waves 3 to wave 4 at the input face $z=0$ is called phase-conjugate reflectivity, R_{pc}.

The build-up of oscillation waves in the cavity can be explained in the following way: the pump wave 2 is scattered from the optical imperfections inside the crystal. A part of this scattered radiation is reflected by the conventional mirror M back into the crystal and interferes with the pump wave 1 to generate a first grating with a low contrast. This grating diffracts wave 2 into wave 3 that interferes with pump wave 2 and generate another grating with the same period and orientation as the previous one but spatially shifted. Wave 3 reflected by the mirror M contributes to create a grating with a higher contrast in the photorefractive crystal. Step by step, waves 3 and 4 gain intensity and the grating amplitudes increases until the process saturates. To be within the single grating approximation all interacting beams (1–4) must record only one grating in the crystal. This is achieved by appropriate choice of the beam paths to ensure the coherence between the pairs of beams (1, 3) and (2, 4) only. This prevents the formation of all index grating except for only one reflection grating that couples the oscillation wave 4 to the pump wave 2 and the oscillation wave 3 to the pump wave 1. The four waves mixing can also be interpreted in terms of real time hologram where waves 1 and 4 write a hologram, wave 2 reads the hologram and is diffracted into wave 3. These two last waves record their own grating which is out of phase with respect to the grating written by waves 1 and 4. For this reason an imbalance between the pump wave intensities is needed for the oscillation buildup otherwise the seed beams will not initiate the process.

2.2. Coupled-wave equations and its general solution

The system of dynamic equations describing the process of nonlinear interaction of electromagnetic waves within a plane wave approximation for slowly varying amplitudes that governs the four-wave mixing in the photorefractive medium of thickness l is of the form [7,8,23]

$$\frac{\partial E_1^*}{\partial z} = \upsilon^* E_3^* \quad (1)$$

$$\frac{\partial E_2}{\partial z} = \upsilon^* E_4 \quad (2)$$

$$\frac{\partial E_3}{\partial z} = \upsilon^* E_1 \quad (3)$$

$$\frac{\partial E_4^*}{\partial z} = \upsilon^* E_2^* \quad (4)$$

$$\tau \frac{\partial \upsilon}{\partial t} + \upsilon = \frac{\gamma}{I_0}(E_1 E_3^* + E_2^* E_4) \quad (5)$$

with the purely real beam coupling coefficient

$$\gamma = \frac{\gamma_0}{1+(\Omega\tau)^2} \tag{6}$$

For a medium with purely non-local response, the non-linear beam coupling coefficient γ is real (Imaginary $\gamma = 0$), where, $\Omega = \omega_4 - \omega_1 (\omega_1 \equiv \omega_2, \omega_3 \equiv \omega_4)$ is a possible frequency detuning of the oscillation wave with respect to the fixed frequency of the pump waves. Eq. (5) describes the temporal variation of the grating amplitude υ, γ_0 being the degenerate energy coupling strength of the interacting beams for $\Omega = 0$, z is the coordinate along the propagation axis, E_i are the slowly varying amplitudes of the electromagnetic waves $i = 1, 2, 3, 4$; E_i^* are their complex conjugate, $I_i = |E_i|^2$ is the intensity of the ith wave, τ is the response time of the photorefractive medium which is inversely proportional to photoconductivity (σ_p) as given by the relation [24]

$$\tau = \frac{\varepsilon\varepsilon_0}{\sigma_p} \tag{7}$$

where ε_0 is the permittivity of free space (8.86×10^{12} C^2/Nm^2), ε is the dielectric constant of the material [25] and I_0 is the total intensity of all four waves is given by,

$$I_0 = I_1 + I_2 + I_3 + I_4 \equiv |E_1|^2 + |E_2|^2 + |E_3|^2 + |E_4|^2 \tag{8}$$

Using Eqs. (6) and (7), the non-linear energy coupling coefficient γ of the interacting beams can be written as,

$$\gamma = \frac{\gamma_0 \sigma_p^2}{\sigma_p^2 + \Omega^2\varepsilon^2} \tag{9}$$

The boundary conditions for the considered semilinear resonator are given as,

$$E_1(0, t) = E_1^0 = const. \tag{10a}$$

$$E_2(l, t) = E_2^1 = const. \tag{10b}$$

$$E_3(0, t) = 0 \tag{10c}$$

$$E_4(l, t) = \sqrt{R} E_3(l, t) \tag{10d}$$

where R is the intensity reflectivity of the feedback mirror (i.e., conventional mirror) M and l is the crystal thickness. The dynamics of the system depends on parameters like input pump intensity ratio $p = I_1(0)/I_2(l)$, frequency detuning (Ω) of the oscillating waves with respect to the fixed frequency of the pump waves, degenerate energy coupling coefficient γ_0 of the interacting beams, dielectric constant (ε) and photoconductivity (σ_p) of the photorefractive materials are the parameters of our considered resonating system. In order to find out the general solution of the coupled Eqs. (1)–(5), we assume the interference term as,

$$g(z) = 2(E_1^* E_3 + E_2 E_4^*) \tag{11}$$

Now, applying the condition (Eq. (11)) into Eqs. (1)–(5) one obtains,

$$\frac{\partial g}{\partial z} = 2\gamma g \tag{12}$$

Eq. (12) can be integrating to yield the expression for interference term g as,

$$g = g_0 \exp(2\gamma z) \quad \text{with} \quad g_0 = g(0) \tag{13}$$

Using Eqs. (11), (13) and (1)–(4) give the following coupled differential equations,

$$\frac{\partial E_1^*}{\partial z} = \frac{\gamma g_0 \exp(2\gamma z)}{I_0} E_3^* \tag{14a}$$

$$\frac{\partial E_2}{\partial z} = \frac{\gamma g_0 \exp(2\gamma z)}{I_0} E_4 \tag{14b}$$

$$\frac{\partial E_3}{\partial z} = \frac{\gamma g_0 \exp(2\gamma z)}{I_0} E_1 \tag{14c}$$

$$\frac{\partial E_4^*}{\partial z} = \frac{\gamma g_0 \exp(2\gamma z)}{I_0} E_2^* \tag{14d}$$

Eqs. (14a)–(14d) yields the expression for the total intensity I_0 of all four waves in terms of interference parameter g_0 as,

$$I_0 = \sqrt{(4|g_0|^2 \exp\{2(\gamma+\gamma^*)z\} + K)} \tag{15}$$

where K is a constant of integration. Eq. (14) can then be completely decoupled by combining Eq. (14a) with Eq. (14c) and Eq. (14b) with Eq. (14d) to give the following differential equations as,

$$\frac{\partial^2 E_1^*}{\partial z^2} - \left[2\gamma - \frac{4|g_0|^2(\gamma+\gamma^*)\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}\right]\frac{\partial E_1^*}{\partial z} - \frac{|g_0\gamma|^2\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}E_1^* = 0 \tag{16a}$$

$$\frac{\partial^2 E_2}{\partial z^2} - \left[2\gamma - \frac{4|g_0|^2(\gamma+\gamma^*)\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}\right]\frac{\partial E_2}{\partial z} - \frac{|g_0\gamma|^2\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}E_2 = 0 \tag{16b}$$

$$\frac{\partial^2 E_3}{\partial z^2} - \left[2\gamma - \frac{4|g_0|^2(\gamma+\gamma^*)\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}\right]\frac{\partial E_3}{\partial z} - \frac{|g_0\gamma|^2\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}E_3 = 0 \tag{16c}$$

$$\frac{\partial^2 E_4^*}{\partial z^2} - \left[2\gamma - \frac{4|g_0|^2(\gamma+\gamma^*)\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}\right]\frac{\partial E_4^*}{\partial z} - \frac{|g_0\gamma|^2\exp\{2(\gamma+\gamma^*)z\}}{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}E_4^* = 0 \tag{16d}$$

For $\gamma + \gamma^* = 0$, corresponding to zero spatial phase shifts, these Eqs. (16a)–(16d) have constant coefficients and can be solved by change of variable method. For this purpose, we define a new variable U as,

$$\frac{\gamma \exp(2\gamma z)}{I_0} = dU \tag{17}$$

Substituting the value of I_0 from Eq. (15) into Eq. (17) and integrating it we get,

$$U = \frac{1}{g_0}\log_e\left[\frac{4|g_0|\exp(2\gamma z) + \sqrt{4|g_0|^2\exp\{2(\gamma+\gamma^*)z\}+K}}{4|g_0| + \sqrt{4|g_0|^2+K}}\right] \tag{18}$$

Using Eqs. (18) and (14a)–(14d) one gets the following differential equations,

$$\frac{\partial E_1^*}{\partial U} = gE_3^* \tag{19a}$$

$$\frac{\partial E_2}{\partial U} = gE_4 \tag{19b}$$

$$\frac{\partial E_3}{\partial U} = gE_1 \tag{19c}$$

$$\frac{\partial E_4^*}{\partial U} = gE_2^* \tag{19d}$$

For photorefractive crystal that operates by diffusion only $\phi = \pi/2$, the non-linear beam coupling constant γ is real. Eqs. (19a)–(19d) can be integrated to yield the expression for the field amplitudes $E_1(U)$, $E_2(U)$, $E_3(U)$ and $E_4(U)$ of the electromagnetic waves as,

$$E_1(U) = \frac{\sqrt{g_0}[((E_4(l)/\sqrt{g_0^*}) + (E_1(0)/\sqrt{g_0})\exp(-|g_0|U(l)))\exp(|g_0|U) + ((E_1(0)/\sqrt{g_0})\exp(|g_0|U(l)) - (E_4(l)/\sqrt{g_0^*}))\exp(-|g_0|U)]}{\exp(|g_0|U(l)) + \exp(-|g_0|U(l))} \quad (20)$$

$$E_2(U) = \frac{\exp(|g_0|U) + \exp(-|g_0|U)}{\exp(|g_0|U(l)) + \exp(-|g_0|U(l))} \quad (21)$$

$$E_3(U) = \sqrt{\left(\frac{g_0}{g_0^*}\right)} E_2(l)\frac{\exp(|g_0|U) + \exp(-|g_0|U)}{\exp(|g_0|U(l)) + \exp(-|g_0|U(l))} \quad (22)$$

$$E_4(U) = \frac{\sqrt{g_0^*}[((E_4(l)/\sqrt{g_0^*}) + (E_1(0)/\sqrt{g_0})\exp(-|g_0|U(l)))\exp(|g_0|U) + ((E_1(0)/\sqrt{g_0})\exp(|g_0|U(l)) - (E_4(l)/\sqrt{g_0^*}))\exp(-|g_0|U)]}{\exp(|g_0|U(l)) + \exp(-|g_0|U(l))} \quad (23)$$

where $E_3(0) = 0$, $E_1(0)$, $E_2(l)$ and $E_4(l)$ are known. Eqs. (21)–(24) represent the general solutions of the coupled Eqs. (1)–(5) for the considered semilinear photorefractive resonator in the photorefractive materials.

2.3. Analysis of self-reproduction of oscillating waves in the semilinear resonators

In this section, we have studied the conditions for which self oscillation occurred in the semilinear resonator. First, we will derive the expression for the phase-conjugate reflectivity of the pumped crystal, which is one of the most important factors that responsible for the self oscillation in the considered resonators. To ensure a stable coherent oscillation two conditions must be met: after each complete round trip of the cavity (1) all losses must be compensated for and (2) the phase of the oscillation wave must return exactly to its initial value (modulo 2π). These conditions are usually called amplitude and phase conditions of oscillation [26,27]. For an oscillator with a phase conjugate mirror two round trips are necessary to ensure that the optical field reproduces its initial state [27]. The steady state oscillation (i.e., self oscillation) occurs if the intensity of the oscillation wave remains the same after one round trip inside the cavity, which leads to condition

$$RR_{pc} = 1 \quad (24)$$

This means that oscillation should occur if phase-conjugate reflectivity overpasses $R_{pc} \geq 1/R$. The phase-conjugate reflectivity of the pumped crystal can be defined as,

$$R_{pc} = \frac{I_3(0)}{I_4(0)} \quad (25)$$

Substituting the values of $I_3(0)$ and $I_4(0)$ from Eqs. (23) and (24) into Eq. (25) we get,

$$R_{pc} = \frac{I_1(0)}{I_4(0)}\tanh^2|g_0|U \quad (26)$$

Eq. (26) represents the exact solution for the reflectivity of a phase-conjugate mirror with a $\phi = \pi/2$ shifted reflection grating. To evaluate $|g_0|U(l)$ we substitute Eqs. (20)–(23) into Eq. (14), using the definition of g, and evaluate at $z = 0$ and l to obtain two equations in g_0, $|g_0|$ and $|g_0|U(l)$. Solving for g_0, taking the magnitude, and then eliminating $|g_0|$ yields

$$\sinh|g_0|U(l) = \frac{\sqrt{I_1(0)I_4(0)}(1 \pm \exp(-\gamma_0 l\,\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2)))}{I_1(0) + (I_1(0) + I_4(0))\exp(-\gamma_0 l\,\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2))} \quad (27)$$

Or,

$$U(l) = \frac{1}{g_0}\sinh^{-1}\left[\frac{\sqrt{(1+p)q}[1 \pm \exp(-\gamma_0 l\,\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2))]}{p + \exp(-\gamma_0 l\,\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2)) + (1+p)q}\right] \quad (28)$$

where $q = I_4(0)/(I_1(0) + I_2(l))$ is the normalized oscillation intensity and $p = I_1(0)/I_2(l)$ is the input pump intensity ratio. In the undepleted pump approximation, we assume that

$$|E_1|^2, |E_2|^2 \gg |E_3|^2, |E_4|^2 \quad (29)$$

Applying the boundary conditions (Eq. (10)) into Eqs. (23)–(28) and using the approximations (Eq. (29)) one gets the following expression for the phase-conjugate reflectivity R_{pc} of pumped crystal as,

$$R_{pc} = \frac{\sinh^2(\gamma_0 l\,\sigma_p^2/2(\sigma_p^2 + \Omega^2\varepsilon^2))}{\cosh^2((\gamma_0 l\,\sigma_p^2/2(\sigma_p^2 + \Omega^2\varepsilon^2)) + (\log_e p/2))} \quad (30)$$

Eq. (30) represents the reflectivity of the phase-conjugate mirror for the non-degenerate four-wave mixing in the undepleted pump approximation [13,28]. In the case of small oscillation intensity $q \to 0$ the undepleted pump approximation is valid and Eqs. (26) and (28) are reduced to Eq. (30). For frequency non-degenerate interaction with pump intensity ratio

$$p = \exp\left(-\frac{\gamma_0 l\sigma_p^2}{\sigma_p^2 + \Omega^2\varepsilon^2}\right) \quad (31)$$

optimized to reach the highest phase-conjugate reflectivity, Eq. (30) reduces to

$$R_{pc} = \sinh^2\left(\frac{\gamma_0 l\sigma_p^2}{2(\sigma_p^2 + \Omega^2\varepsilon^2)}\right) \quad (32)$$

and predicts, for high-enough energy coupling strengths of the interacting beams, an exponential growth of the phase-conjugate reflectivity

$$R_{pc} \approx \frac{\exp(-\gamma_0 l\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2))}{4} \quad (33)$$

Using Eq. (33), Eq. (24) takes the form

$$\exp\left(-\frac{\gamma_0 l\sigma_p^2}{\sigma_p^2 + \Omega^2\varepsilon^2}\right) \approx \frac{4}{R} \quad (34)$$

With $-\ln(R)$ being the dimensionless losses because of mirror transparency the oscillation condition becomes

$$\left(-\frac{\gamma_0 l\sigma_p^2}{\sigma_p^2 + \Omega^2\varepsilon^2}\right)_{th} = -\ln(R) + \ln 4 \quad (35)$$

Eq. (35) resembles, to a certain extent, the oscillation condition for conventional lasers, which says that, to get the oscillation, the exponential gain should compensate for all types of cavity losses [6]. The necessity to ensure R_{pc} at least larger than unity to achieve the oscillation leads to the additional term, ln 4, on the right-hand side of Eq. (35). This means that with a properly selected degenerate energy coupling coefficient γ_0, crystal thickness l, frequency detuning Ω, dielectric constant ε and photoconductivity σ_p, the amplified reflectivity of a phase conjugate mirror compensates for all cavity losses and the waves 3 and 4 self-develop spontaneously; i.e., the oscillation occurs.

2.4. Threshold and output characteristics of the resonator

2.4.1. Intensity of oscillations

In this section we will find out the expression for the intensity of oscillation and existence of the oscillation intensity of a semilinear optical resonator when the control parameters vary. For this, we are applying the condition ($I_4(0)=RI_3(0)$) into the Eqs. (24), (26) and (28) with a new variable $s=I_3(0)/I_1(0)$ we have,

$$R^2p^2s^2+Rps\left\{2\left[p+\exp\left(-\frac{\gamma_0\sigma_p^2 l}{\sigma_p^2+\Omega^2\varepsilon^2}\right)\right]+\left[1\pm\exp\left(-\frac{\gamma_0\sigma_p^2 l}{\sigma_p^2+\Omega^2\varepsilon^2}\right)\right]^2\right\}+\left[p+\exp\left(-\frac{\gamma_0\sigma_p^2 l}{\sigma_p^2+\Omega^2\varepsilon^2}\right)\right]^2-Rp\left[1\pm\exp\left(-\frac{\gamma_0\sigma_p^2 l}{\sigma_p^2+\Omega^2\varepsilon^2}\right)\right]^2=0 \quad (36)$$

There are two possible solutions of Eq. (36) we take only positive solution

$$s=\frac{-2[p+\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]-[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2+[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]\sqrt{[1+\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2+4p(R+1)}}{2Rp} \quad (37)$$

Eq. (37) represents the expression for the normalized oscillation intensity of a semilinear photorefractive optical resonator due to non-degenerate four-wave mixing in the photorefractive materials.

2.4.2. Output characteristics

The aim of this section is that to study the output characteristics and performance of a semilinear optical resonator. The degenerate energy coupling strength ($\gamma_0 l$) of the interacting beams is one of the most important parameter in order to establishing the threshold condition for the semilinear optical resonator. At the threshold i.e., for $s=0$ Eq. (36) gives the known threshold condition

$$(\gamma_0 l)_{th}=-\log_e\left[\frac{\sqrt{p}(\sqrt{R}-\sqrt{p})(\sigma_p^2+\Omega_{th}^2\varepsilon^2)}{\sigma_p^2(\sqrt{Rp}+1)}\right] \quad (38)$$

The dependence of the saturation oscillation at large degenerate energy coupling strength ($\gamma_0 l$) calculated from Eq. (37) we get

$$\lim_{\gamma_0 l\to\infty} s=\frac{-2p-1+\sqrt{1+4p(R+1)}}{2Rp} \quad (39)$$

The oscillation is possible within a certain range of the pump ratio $[p_{th1}, p_{th2}]$, which depends on the degenerate energy coupling strength ($\gamma_0 l$) of the interacting beams and the conventional mirror reflectivity (R). Again, in the vicinity of the threshold, the oscillation intensity increases with the pump ratio which suggests a supercritical bifurcation [23]. One should look therefore for a reason for the bifurcation in the oscillation spectrum that is related to gain, i.e., to spectra of amplified phase conjugate reflectivity. The maximum of the oscillation intensity is calculated from $\partial s/\partial p=0$. It is easy to show that the maximum value

$$(s)_{\max}=\frac{R[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2-4\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))}{R[1+\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2} \quad (40)$$

is reached at

$$p_{\max}=\frac{\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))[1+\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2}{(1+R)[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2} \quad (41)$$

one interesting characteristics of the considered oscillators should be noted that the pump ratio at which the largest oscillation intensity is reached does not coincide with that which provides the smallest threshold value of the coupling strength. This is in contrast to conventional lasers where the oscillation intensity is linearly proportional to the over-threshold pumping [6].

2.4.3. Analysis of threshold conditions for the oscillations

In this section we have investigate the threshold behavior of the oscillation intensity and the maximum frequency detuning effect (i.e., non-degeneracy) for a semilinear optical resonator. For this purpose the same condition of oscillation (Eq. (24)) is considered, with low-signal phase-conjugate reflectivity given by Eq. (30), which is well justified at the threshold of oscillation. The distinction is that the oscillation wave is allowed to be shifted in frequency to Ω with respect to the pump waves, i.e., $\omega_1=\omega_2=\omega_{pump}$, whereas $\omega_3=\omega_{pump}\pm\Omega$ and $\omega_4=\omega_{pump}\mp\Omega$. It is known [29,30] that, being reflected from the considered phase-conjugate mirror, the light wave with frequency ($\omega_{pump}+\Omega$) acquires frequency ($\omega_{pump}-\Omega$) and vice versa. This is one of the reasons why the oscillation wave is self-reproduced only after two full round trips in the cavity, as distinct from conventional lasers where one round trip is sufficient [6]. Therefore for the non-degenerate case, the oscillation wave should always contain two components, shifted symmetrically by $\pm\Omega$ with respect to the pump frequency. We find an approximate solution for $I_3(0)$ verses $\gamma_0 l$ or verses p in the close vicinity of the oscillation threshold and to confirm the type of bifurcation. Taking p as a variable, the threshold values $p_{th\,1,2}$ are obtained by solving Eq. (36) with $s=0$

$$p_{th1,2}=\frac{R[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2-2\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))\mp R[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]\sqrt{[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2-4\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))/R}}{2} \quad (42)$$

Then, normalized oscillation intensity, $s=I_3(0)/I_1(0)$, which is defined by Eq. (28) is represented as a series of p in the vicinity of $p_{th1,2}$. We found that the intensity changes linearly with $p-p_{th1,2}$

$$s=\frac{p-p_{th1,2}}{-R\mp((\sqrt{R}[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))](1+R))/\sqrt{R[1-\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))]^2-4\exp(-\gamma_0\sigma_p^2 l/(\sigma_p^2+\Omega^2\varepsilon^2))})} \quad (43)$$

The denominator is positive for p_{th1} and negative for p_{th2} for all values of the non-linear energy coupling strength and conventional mirror reflectivity. This means that the absolute value of the oscillation amplitude bifurcates supercritically in both cases [23]. A similar analysis of the threshold with respect to the coupling strength $\gamma_0 l$ as a variable leads to the expression

$$s \approx \frac{2\sqrt{p}(p+1)\sigma_p^2}{\exp(((\gamma_0 l)_{th}\sigma_p^2)/(\sigma_p^2+\Omega_{th}^2\varepsilon^2))(2p+1)\sqrt{R}} \times \left(\frac{\gamma_0 l}{\sigma_p^2+\Omega^2\varepsilon^2} - \frac{(\gamma_0 l)_{th}}{\sigma_p^2+\Omega_{th}^2\varepsilon^2}\right) \quad (44)$$

Again, the intensity is linearly increasing with the coupling strength and the coefficient is negative for any values of p and R. Taking into the account that Eq. (44) is derived for the oscillator geometry with a negative non-linear degenerate energy coupling strength. This means that the absolute value of the oscillation amplitude bifurcates from zero value supercritically [8,23].

3. Results and discussion

The phase-conjugate reflectivity (R_{pc}) is an important factor responsible for the self oscillation in the semilinear optical resonator [6,12,28]. This phase-conjugate reflectivity (Eq. (30)) is also depends on the frequency detuning (Ω) of the oscillation beam with respect to the fixed frequency of the pump waves, degenerate energy coupling strength ($\gamma_0 l$) of the interacting beams, input pump intensity ratio (p), dielectric constant (ε) and photoconductivity (σ_p) of the photorefractive materials The resonator with one phase-conjugate mirror has the ability to compensate for intra-cavity phase distortions. This may be of practical importance for hybrid systems with the additional laser amplifiers inside the cavity with high gain but moderate optical quality [31,32]. Variations of phase-conjugate reflectivity (R_{pc}) with frequency detuning (Ω) for different values of σ_p (fixed ε = 10, $\gamma_0 l$ = 10 and p = 1), $\gamma_0 l$ (fixed ε = 10, σ_p = 1.0 pS/cm and p = 1), p (fixed ε = 10, $\gamma_0 l$ = 10 and σ_p = 1.0 pS/cm) and ε (fixed p = 1, $\gamma_0 l$ = 10 and σ_p = 1.0 pS/cm) are shown in Fig. 2(a), (b), (c) and (d) respectively.

From Fig. 2(a) it is obvious that the reflectivity (R_{pc}) of the phase-conjugate mirror increases with increasing the frequency detuning (Ω) of the oscillation beam and reaches to maximum values after certain region constant around Ω = 0 it decreases with increasing the frequency detuning of the oscillation beam. It means that the steady state oscillation occurs over this constant region around Ω = 0. This happens because of the losses of the conventional mirror is exactly compensated by the gain of the phase-conjugate mirror. Similar variations of R_{pc} with $\gamma_0 l$, p and ε can be seen from Fig. 2(b), (c) and (d) respectively. From Fig. 2(a), it is interesting to note that for the lower value of photoconductivity σ_p(<0.1 pS/cm) of the photorefractive crystal the steady sate oscillation occurs only when the frequency detuning is limited to small region around Ω = 0 whereas for higher values of σ_p(=2.0 pS/cm) the semilinear optical resonator can oscillate at almost any frequency detuning Ω of the oscillation beam. But reverse of the case is found for ε, which can be seen from Fig. 2(d). However, it is also noted from Fig. 2(c) that for different pump intensity ratios (p) of the input beams, the phase-conjugate reflectivity of the pumped crystal is found to be higher and the region of steady state oscillations is found to be small around Ω = 0 as compared to the other cases (i.e., $\gamma_0 l$, ε and σ_p). Thus, one can conclude that the largest phase-conjugate reflectivity is reached at Ω = 0 for relatively small values of pump intensity ratio (p < 0.2) of the input beams and vice versa.

Fig. 3(a), (b), (c) and (d) presents the variations of phase-conjugate reflectivity (R_{pc}) with photoconductivity σ_p (in pS/cm) for different values of Ω (fixed ε = 10, $\gamma_0 l$ = 10 and p = 1), ε (fixed $\gamma_0 l$ = 10, Ω = 0.2 Hz and p = 1), $\gamma_0 l$ (fixed ε = 10, Ω = 0.2 Hz and p = 1) and p (fixed Ω = 0.2 Hz, $\gamma_0 l$ = 10 and ε = 10) respectively. From Fig. 3(a), it can be seen that the reflectivity of the phase-conjugate mirrors increases exponentially with photoconductivity of the photorefractive crystals and finally attains a saturation value. It is interesting to note that the saturation value of R_{pc} decreases with increasing frequency detuning (Ω) of the oscillating beam with

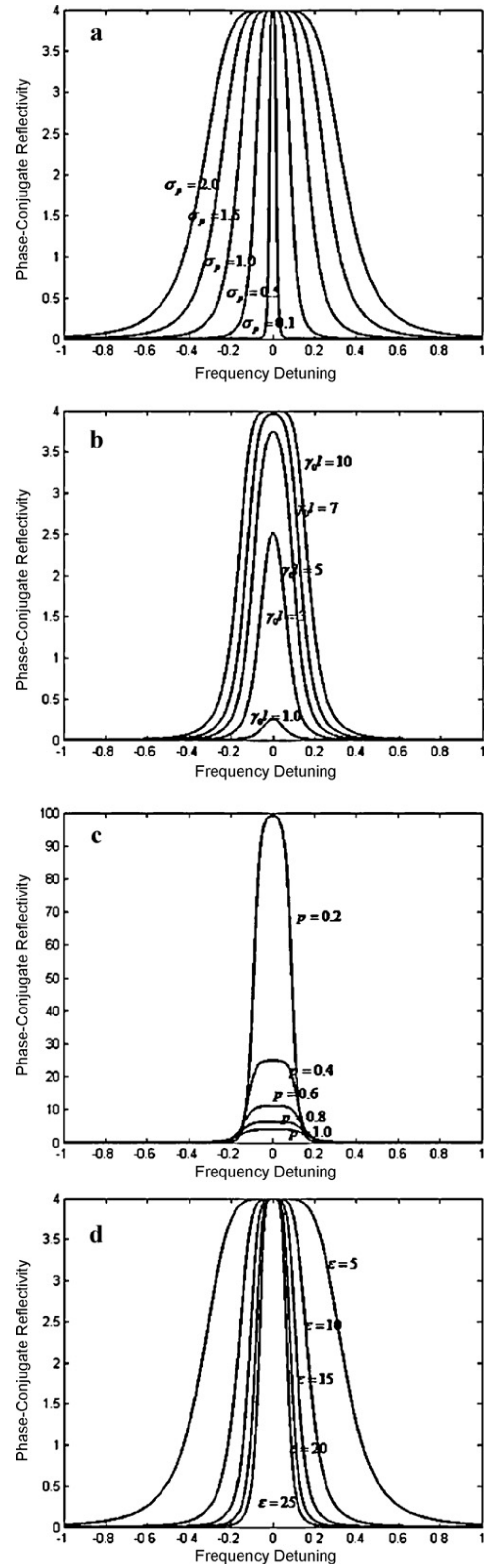

Fig. 2. Variations of phase-conjugate reflectivity 'R_{pc}' with frequency detuning 'Ω' for different values of (a) σ_p (fixed ε = 10, $\gamma_0 l$ = 10 and p = 1); (b) $\gamma_0 l$ (fixed ε = 10, σ_p = 1.0 pS/cm and p = 1); (c) p (fixed ε = 10, $\gamma_0 l$ = 10 and σ_p = 1.0 pS/cm) and (d) ε (fixed p = 1, $\gamma_0 l$ = 10 and σ_p = 1.0 pS/cm) respectively.

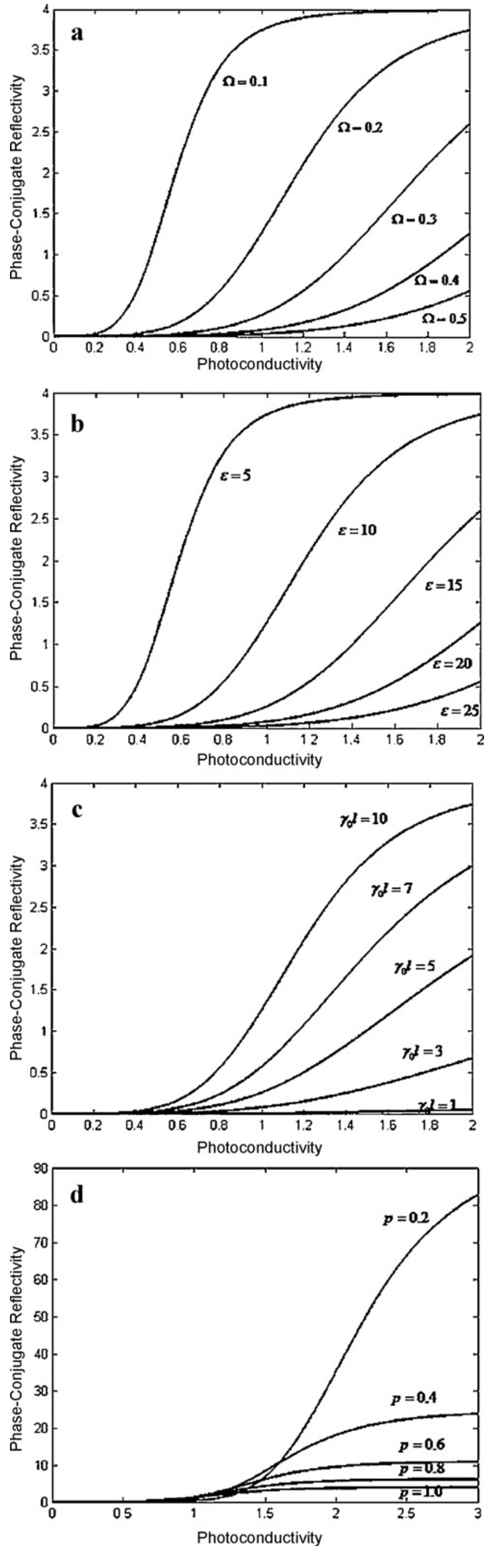

Fig. 3. Variations of phase-conjugate reflectivity 'R_{pc}' with photoconductivity 'σ_p' for different values of (a) Ω (fixed $\varepsilon = 10$, $\gamma_0 l = 10$ and $p = 1$); (b) ε (fixed $\gamma_0 l = 10$, $\Omega = 0.2$ Hz and $p = 1$); (c) $\gamma_0 l$ (fixed $\varepsilon = 10$, $\Omega = 0.2$ Hz and $p = 1$) and (d) p (fixed $\Omega = 0.2$ Hz, $\gamma_0 l = 10$ and $\varepsilon = 10$) respectively.

respect to the fixed frequency of the pump waves. Similar variations of R_{pc} with ε, $\gamma_0 l$ and p can be seen from Fig. 3(b), (c) and (d) respectively. However, for a given value of σ_p, the higher value of phase-conjugate reflectivity of the pumped crystal is found to be occurred at much lower value of pump intensity ratio ($p < 0.20$) of the input beams (Fig. 3(d)) as compared to the other parameters i.e., ε, $\gamma_0 l$ and Ω (Fig. 3(a)–(c)).

Fig. 4(a), (b) (c) and (d) respectively shows the variations of phase-conjugate reflectivity (R_{pc}) with dielectric constant (ε) for different values of Ω (fixed $\sigma_p = 1.0$, $\gamma_0 l = 10$ and $p = 1$), p (fixed $\gamma_0 l = 10$, $\Omega = 0.2$ Hz and $\sigma_p = 1.0$ pS/cm), $\gamma_0 l$ (fixed $\sigma_p = 1.0$, $\Omega = 0.2$ Hz and $p = 1$) and σ_p (fixed $\Omega = 0.2$ Hz, $\gamma_0 l = 10$ and $p = 1$). Fig. 4(a), it is clear that for different values of frequency detuning, phase-conjugate reflectivity of the pumped crystal decays drastically with increasing ε. Similar variations of phase-conjugate reflectivity (R_{pc}) of the pumped crystal with p, $\gamma_0 l$ and σ_p can be seen from Fig. 4(b), (c) and (d) respectively. It is to be noted that for a given value of dielectric constant, there is fall in phase-conjugate reflectivity of the pumped crystal is more for higher value of frequency detuning of the oscillating beam (Fig. 4(a)). But reverse of the case is found for σ_p, which can be seen from Fig. 4(d). It can also be seen that for a given value of dielectric constant of the photorefractive crystals, the higher value of phase-conjugate reflectivity of the pumped crystal can be achieved at much lower value of pump intensity ratio ($p < 0.2$) of the input beams (Fig. 4(b)) as compared to the others parameters i.e., Ω, $\gamma_0 l$ and σ_p (Fig. 4(a), (c) and (d)).

Due to non-degenerate four wave-mixing in the photorefractive materials, the intensity of oscillation (s) inside the semilinear resonator depends on the degenerate energy coupling strength ($\gamma_0 l$) of the interacting beams, frequency detuning (Ω) of the oscillating beam with respect to the fixed frequency of the pump waves, pump intensity ratio (p) of the input beams, conventional mirror reflectivity (R), dielectric constant (ε) and photoconductivity (σ_p) of the photorefractive materials (Eq. (37)). The variations of normalized oscillation intensity (s) with frequency detuning (Ω) for different values of σ_p (fixed $R = 1$, $\varepsilon = 10$, $\gamma_0 l = 10$ and $p = 1$), ε (fixed $R = 1$, $\gamma_0 l = 10$, $\sigma_p = 1.0$ pS/cm and $p = 1$), $\gamma_0 l$ (fixed $R = 1$, $\sigma_p = 1.0$ pS/cm, $\varepsilon = 10$ and $p = 1$), p (fixed $R = 1$, $\sigma_p = 1.0$ pS/cm, $\varepsilon = 10$ and $\gamma_0 l = 10$) and R (fixed $\varepsilon = 10$, $\gamma_0 l = 10$, $\sigma_p = 1.0$ pS/cm and $p = 1$) are shown in Fig. 5(a), (b), (c), (d) and (e) respectively. It can been (Fig. 5(a)) that for fixed R, ε, $\gamma_0 l$ and p, the intensity of oscillation s increases with increasing the frequency detuning (Ω) [i.e., frequency shift] of the oscillating beam and reaches to maximum values after certain region constant around $\Omega = 0$ it decreases with increasing the frequency detuning of the oscillation beam. It means that resonator can oscillate over this constant region around $\Omega = 0$. This happens because of the losses of the conventional mirror is exactly compensated by the gain of the phase-conjugate mirror. Similar variations of oscillation intensity (s) with ε, $\gamma_0 l$, p and R can be seen from Fig. 5(b), (c), (d) and (e) respectively. But reverse of the case is found for ε, which can be seen from Fig. 5(d). From Fig. 5(a), it is interesting to note that for the higher value of photoconductivity σ_p(=2.0 pS/cm) of the photorefractive crystals the semilinear resonator can oscillate at almost any frequency detuning (Ω) of the oscillation beam with respect to the fixed frequency of the pump waves whereas for the lower value of photoconductivity σ_p(<0.1 pS/cm) oscillation occurs only when the frequency detuning is limited to small region around $\Omega = 0$. But reverse of the case is found for ε, p and R, which can be seen from Fig. 5(b), (d) and (e) respectively. However, it is also noted from Fig. 5(a) and (b) that for a given value of Ω, the magnitude of the steady state oscillations are found to be same with increasing either the value of σ_p or ε whereas the magnitude of the steady state oscillations are found to be increases with increasing the value of $\gamma_0 l$ or p or R (Fig. 5(c)–(e)). From Fig. 5, one may conclude that for a given value of Ω, the region of steady state oscillation can be enhanced by

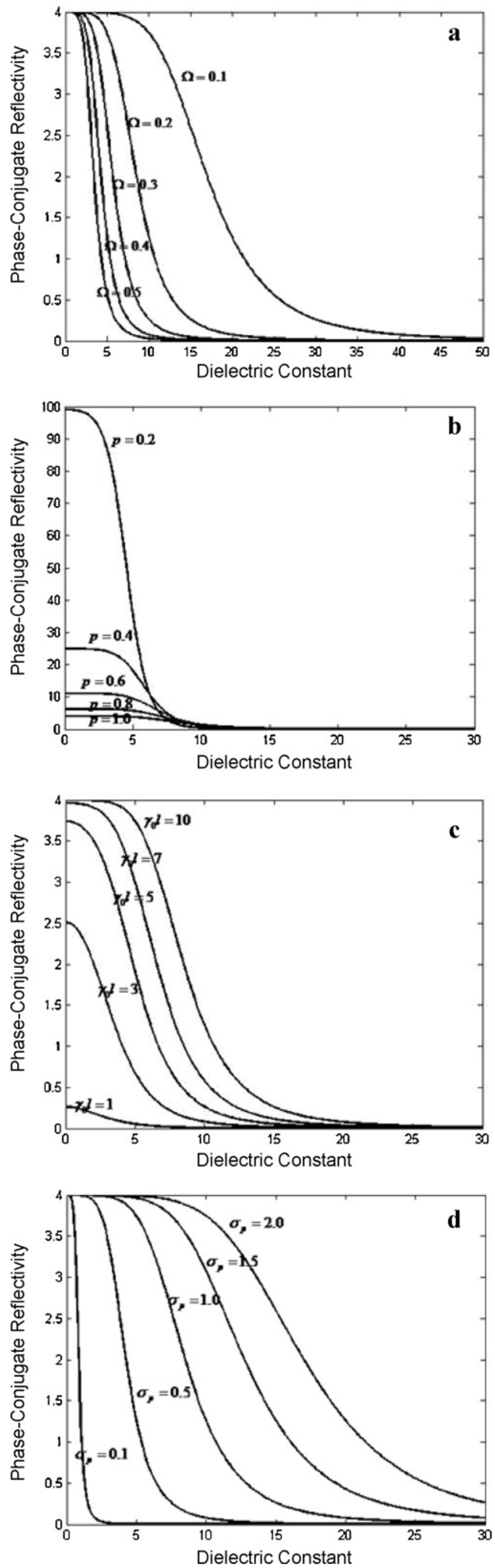

Fig. 4. Variations of phase-conjugate reflectivity 'R_{pc}' with dielectric constant 'ε' for different values of (a) Ω (fixed σ_p = 1.0 pS/cm, $\gamma_0 l$ = 10 and p = 1); (b) p (fixed $\gamma_0 l$ = 10, Ω = 0.2 Hz and σ_p = 1.0 pS/cm); (c) $\gamma_0 l$ (fixed σ_p = 1.0 pS/cm, Ω = 0.2 Hz and p = 1) and (d) σ_p (fixed Ω = 0.2 Hz, $\gamma_0 l$ = 10 and p = 1) respectively.

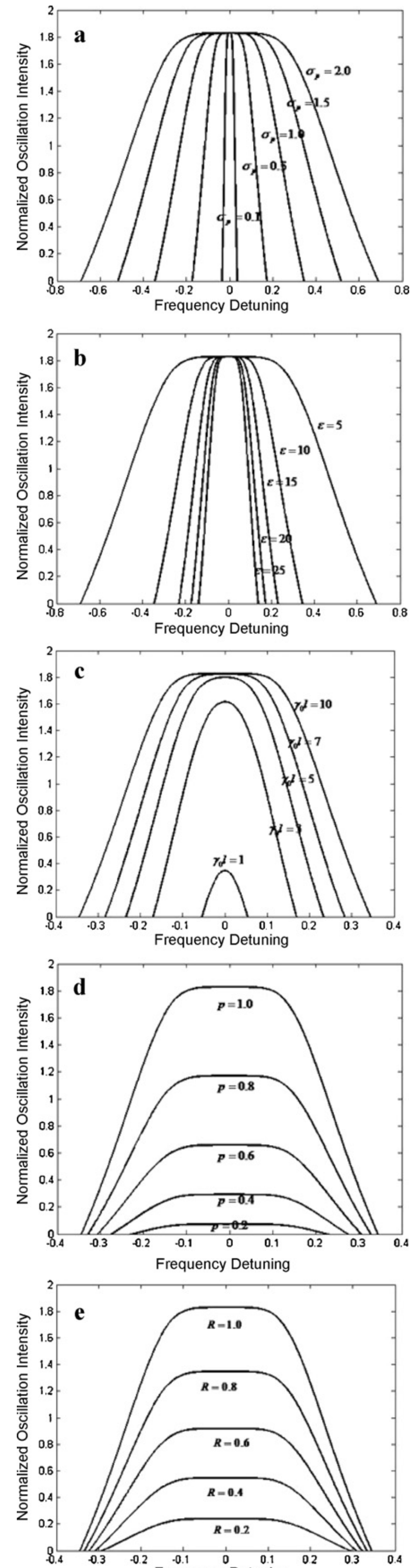

Fig. 5. Variations of normalized oscillation intensity 's' with frequency detuning 'Ω' for different values of (a) σ_p (fixed R = 1, ε = 10, $\gamma_0 l$ = 10 and p = 1); (b) ε (fixed R = 1, $\gamma_0 l$ = 10, σ_p = 1.0 pS/cm and p = 1); (c) $\gamma_0 l$ (fixed R = 1, σ_p = 1.0 pS/cm, ε = 10 and p = 1); (d) p (fixed R = 1, σ_p = 1.0 pS/cm, ε = 10 and $\gamma_0 l$ = 10) and (e) R (fixed ε = 10, $\gamma_0 l$ = 10, σ_p = 1.0 pS/cm and p = 1) respectively.

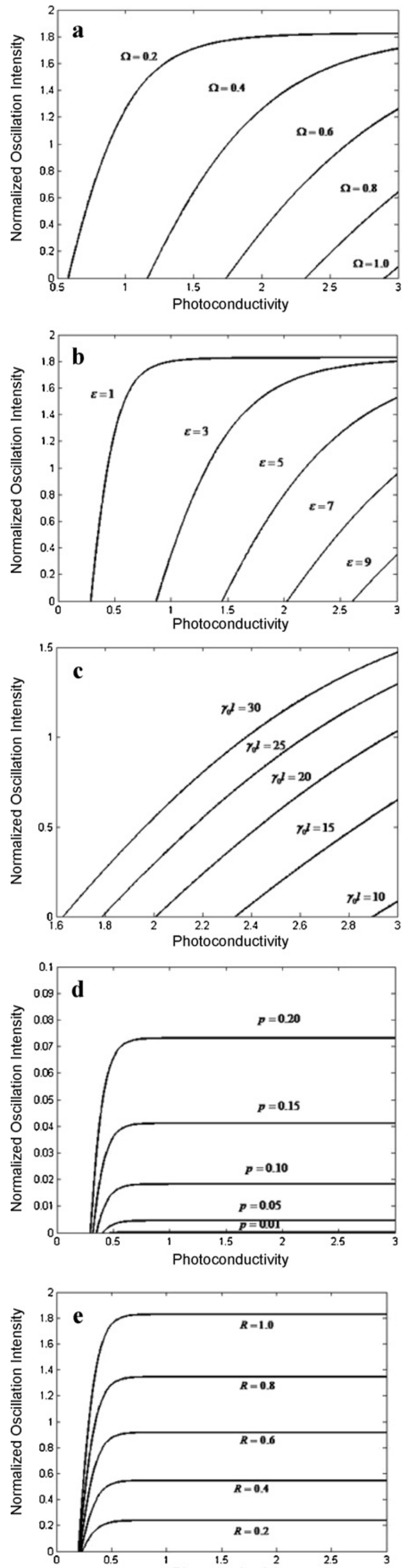

Fig. 6. Variations of normalized oscillation intensity 's' with photoconductivity 'σ_p' for different values of (a) Ω (fixed $R=1$, $\varepsilon=10$, $\gamma_0 l=10$ and $p=1$); (b) ε (fixed $R=1$, $\gamma_0 l=10$, $\Omega=1.0$ Hz and $p=1$); (c) $\gamma_0 l$ (fixed $R=1$, $\Omega=1.0$ Hz, $\varepsilon=10$ and $p=1$); (d) p (fixed $R=1$, $\Omega=0.2$ Hz, $\varepsilon=5$ and $\gamma_0 l=20$) and (e) R (fixed $\varepsilon=5$, $\gamma_0 l=20$, $\Omega=0.2$ Hz and $p=1$) respectively.

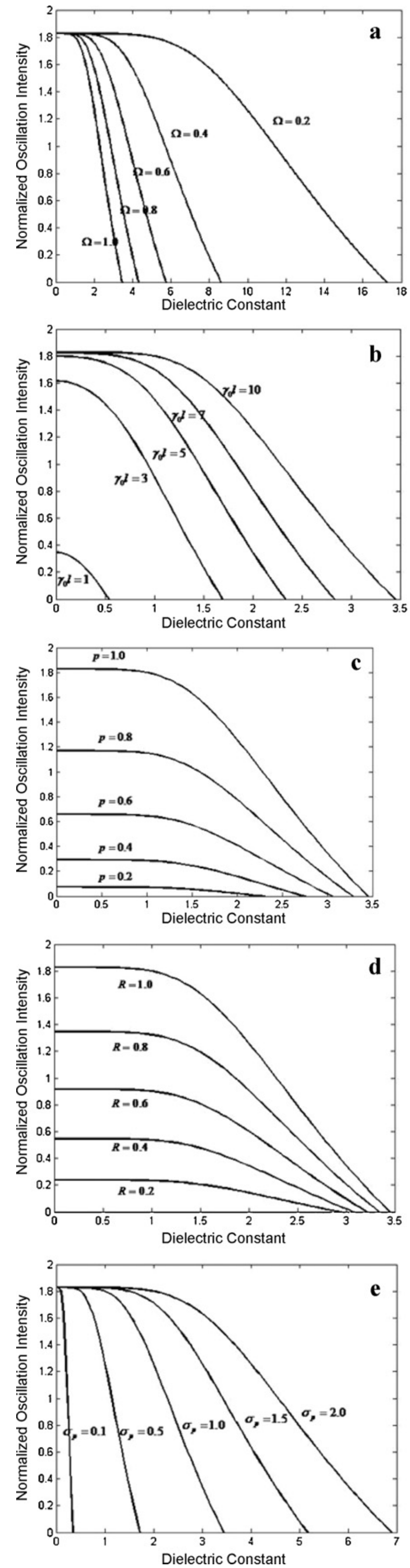

Fig. 7. Variations of normalized oscillation intensity 's' with dielectric constant 'ε' for different values of (a) Ω (fixed $R=1$, $\sigma_p=1.0$ pS/cm, $\gamma_0 l=10$ and $p=1$); (b) $\gamma_0 l$ (fixed $R=1$, $\Omega=1.0$ Hz, $\sigma_p=1.0$ pS/cm and $p=1$); (c) p (fixed $R=1$, $\sigma_p=1.0$ pS/cm, $\gamma_0 l=10$ and $\Omega=1.0$ Hz); (d) R (fixed $\Omega=1.0$ Hz, $\sigma_p=1.0$ pS/cm, $\varepsilon=10$ and $\gamma_0 l=10$) and (e) σ_p (fixed $\Omega=1.0$ Hz, $\gamma_0 l=10$, $R=1$ and $p=1$) respectively.

taking a photorefractive crystal of higher photoconductivity and lower dielectric constant.

Fig. 6(a), (b), (c), (d) and (e) respectively depicts the variations of normalized oscillation intensity (s) with photoconductivity σ_p (in pS/cm) for different values of Ω (fixed $R = 1$, $\varepsilon = 10$, $\gamma_0 l = 10$ and $p = 1$), ε (fixed $R = 1$, $\gamma_0 l = 10$, $\Omega = 1.0$ Hz and $p = 1$), $\gamma_0 l$ (fixed $R = 1$, $\Omega = 1.0$ Hz, $\varepsilon = 10$ and $p = 1$), p (fixed $R = 1$, $\Omega = 0.2$ Hz, $\varepsilon = 5$ and $\gamma_0 l = 20$) and R (fixed $\varepsilon = 5$, $\gamma_0 l = 20$, $\Omega = 0.2$ Hz and $p = 1$). From Fig. 6(a), it can be seen that the normalized oscillation intensity increases rapidly with σ_p of the photorefractive crystals and finally attains a saturation value. It is interesting to note that the saturation value of oscillation intensity s decreases with increasing the frequency detuning (Ω) of the oscillating beam with respect to the fixed frequency of the pump waves. Similar variations of s with ε can be seen from Fig. 6(b). With increasing the value of photoconductivity of the photorefractive crystals, the intensity of oscillation increases linearly (Fig. 6(c)). For a given value of σ_p, the higher value of intensity of oscillation can be achieved at the higher value of energy coupling strength of the interacting beams. Fig. 6(d) and (e), it is found that oscillation starts from zero at the threshold and then increases the photoconductivity (σ_p) until saturation is reached. The results of the calculation show the soft mode of the oscillation onset. The intensity of oscillation beam is zero exactly at threshold and is increasing gradually until the saturation level is reached with increasing photoconductivity of the photorefractive materials. The largest intensity at saturation is reached for highly reflecting conventional mirror $R = 1$ whereas for smaller reflectivity the oscillation intensity decreases even inside the resonator (Fig. 6(e)). It can also be seen from Fig. 6(d) that the intensity of oscillation increases with increase of σ_p for different p but becomes constant after particular value of σ_p and these σ_p values are greater for higher value of p. It is interesting to note that the saturation value of oscillation intensity s increases with R (Fig. 6(e)). It is also observed that for a given value of σ_p, grown in intensity of oscillation is found to be much higher at much lower value of the pump intensity ratio ($p > 0.20$) of the input beams (Fig. 6(d)).

Fig. 7(a), (b), (c), (d) and (e) respectively shows the variations of normalized oscillation intensity 's' with dielectric constant 'ε' for different values of Ω (fixed $R = 1$, $\sigma_p = 1.0$ pS/cm, $\gamma_0 l = 10$ and $p = 1$), $\gamma_0 l$ (fixed $R = 1$, $\Omega = 1.0$ Hz, $\sigma_p = 1.0$ pS/cm and $p = 1$), p (fixed $R = 1$, $\sigma_p = 1.0$ pS/cm, $\gamma_0 l = 10$ and $\Omega = 1.0$ Hz), R (fixed $\Omega = 1.0$ Hz, $\sigma_p = 1.0$ pS/cm, $\varepsilon = 10$ and $\gamma_0 l = 10$) and σ_p (fixed $\Omega = 1.0$ Hz, $\gamma_0 l = 10$, $R = 1$ and $p = 1$). It can be seen (Fig. 7(a)) that for fixed R, σ_p, $\gamma_0 l$ and p, the intensity of oscillation s is remains constant up to the certain value of ε and afterward oscillation intensity decreases with further increasing the dielectric constant of the photorefractive crystals. For a given value of ε, the region of steady state oscillation (i.e., saturation) is higher at much lower value of frequency detuning ($\Omega < 0.2$ Hz) of the oscillating beam with respect to the fixed frequency of the pump waves. Similar variations of oscillation intensity (s) with $\gamma_0 l$, p, R and σ_p can be seen from Fig. 7(b), (c), (d) and (e) respectively. It is to be noted that for different values of σ_p, the region of steady state oscillation is much higher as compared to the region of steady state oscillation for different values of pump intensity ratio (p) of the input beams (Fig. 7(c) and (e)). It can also be seen that for a given value of dielectric constant of the photorefractive crystals, the higher value of oscillation intensity can be achieved at much lower value of photoconductivity (σ_p) as compared to the degenerate energy coupling strength ($\gamma_0 l$) of the interacting beams (Fig. 7(b) and (e)).

4. Conclusion

In the present paper, frequency detuning dependence of four-beam coupling in photorefractive crystal pumped with two counter-propagating waves for a semilinear coherent optical resonator on the oscillation conditions using reflection grating configurations has been studied in the case of non-degenerate-wave mixing .The coherent oscillation in the considered geometry self-develops from the noise and its frequency is not imposed by any specific frequency in the broad and smooth spectrum of the seeding radiation. On the other side, the semilinear oscillator belongs to oscillators with unclosed cavities for which the spectrum of cavity longitudinal modes is discrete but the eigen-frequencies are not fixed [33]. The condition of self reproduction for oscillation wave intensity imposes a condition that the wave intensity should remain the same after two consecutive reflections, once from the conventional mirror and then from the phase conjugate mirror for all types of cavity losses [6]. The phase-conjugate reflectivity is an important factor responsible for the self oscillation in the considered resonators [8,28]. By properly selecting the experimental conditions (pump intensity ratio p, degenerate energy coupling coefficient γ_0 of the interacting beams and frequency detuning Ω of the oscillation beam with respect to the fixed frequency of the pump waves) and photorefractive crystal parameters (crystal thickness l, dielectric constant ε and photoconductivity σ_p), the amplified reflectivity of a phase conjugate mirror compensates for all cavity losses and the waves 3 and 4 self-develop spontaneously; i.e., the oscillation occurs in the resonators. It has been observed that the cavity mode frequency does not affect the oscillation frequency because of the self-adaptable nature of the phase conjugate mirror. For these resonators, it is also found that the conventional mirror of the cavity cannot possess spectral selectivity within the region of several Hz. The phase-conjugate reflectivity, R_{pc}, versus frequency detuning, Ω, curves shows that the largest phase-conjugate reflectivity is reached at $\Omega = 0$ for relatively small values of pump intensity ratio ($p < 0.2$) of the input beams and vice versa .The oscillation intensity, s, versus frequency detuning, Ω, curves depict that for the higher value of photoconductivity σ_p(>2.0 pS/cm) of the photorefractive crystals the semilinear resonator can oscillate at almost any frequency detuning (Ω) of the oscillation beam with respect to the fixed frequency of the pump waves whereas for the lower value of photoconductivity σ_p(<0.1 pS/cm) oscillation occurs only when the frequency detuning is limited to small region around $\Omega = 0$. But reverse of the case is found for dielectric constant (ε), pump intensity ratio (p) and conventional mirror reflectivity (R), which can be seen from the curves s versus Ω.

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References

[1] J. Feinberg, R. Hellwarth, Phase-conjugating mirror with continuous-wave gain, Opt. Lett. 5 (1980) 519–521.
[2] S.R. Liu, G. Indebetouw, Periodic and chaotic spatiotemporal states in a phase-conjugate resonator using a photorefractive $BaTiO_3$ phase-conjugate mirror, J. Opt. Soc. Am. B 9 (1992) 1507–1520.
[3] P. Mathey, P. Jullien, O. Shinkarenko, S. Odoulov, Manifestation of Curie–Weiss law for optical phase transition, Appl. Phys. B 73 (2001) 711–715.
[4] P. Mathey, P. Jullien, S. Odoulov, O. Shinkarenko, Second-order optical phase transition in a semilinear photorefractive oscillator with two counterpropagating pump waves, J. Opt. Soc. Am. B 19 (2001) 405–411.
[5] P. Mathey, S. Odoulov, D. Rytz, Instability of single-frequency operation in semilinear photorefractive coherent oscillators, Phys. Rev. Lett. 89 (2002), 053901-4.
[6] P. Mathey, S. Odoulov, D. Rytz, Oscillation spectra of semilinear photorefractive coherent oscillator with two pump waves, J. Opt. Soc. Am. B 19 (2002) 2967–2977.
[7] M. Grapinet, P. Mathey, S. Odoulov, D. Rytz, Semilinear coherent oscillator with reflection-type photorefractive gratings, Appl. Phys. B 77 (2003) 551–554.

[8] R. Rebhi, P. Mathey, M. Grapinet, H.R. Jauslin, S. Odoulov, Phase mismatch effects on the dynamics of a semilinear photorefractive oscillator with reflection-type gratings, Appl. Phys. B 91 (2008) 583–589;
R. Rebhi, P. Mathey, M. Grapinet, H.R. Jauslin, S. Odoulov, G. Cook, D.R. Evans, Four-wave-mixing coherent oscillator with frequency shifted feedback and misaligned pump waves, Opt. Lett. 34 (2009) 377–379.

[9] B. Sturman, P. Mathey, H.R. Jauslin, S. Odoulov, A. Shumelyuk, Modeling of the photorefractive nonlinear response in $Sn_2P_2S_6$ crystals, J. Opt. Soc. Am. B 24 (2007) 1303–1309.

[10] M. Grapinet, P. Mathey, H.R. Jauslin, B. Sturman, D. Rytz, S. Odoulov, Frequency degenerate and non-degenerate nonlinear regimes for semi-linear photorefractive oscillator, Eur. Phys. J. D 41 (2007) 363–369.

[11] A. Shumelyuk, A. Hryhorashchuk, S. Odoulov, Oscillation dynamics of a $Sn_2P_2S_6$-based semilinear optical oscillator, Ukr. J. Phys. 51 (2006) 547–551.

[12] Y. Uesu, K. Yasukawa, N. Saito, S. Odoulov, K. Shcherbin, A.I. Ryskin, A.S. Shcheulin, Backward-wave four-wave mixing and coherent oscillation in CdF2:Ga,Y, Appl. Phys. B 78 (2004) 601–605.

[13] M. Cronin-Golomb, B. Fischer, J.O. White, A. Yariv, Theory and applications of four-wave mixing in photorefractive media, IEEE J. Quantum Electron. QE-20 (1984) 12–30.

[14] A.A. Bagan, V.B. Gerasimov, A.V. Golyanov, V.E. Ogluzdin, V.A. Sugrobov, I.L. Rubtsova, A.I. Khyzhnjak, Conditions for the stimulated emission from a laser with cavities coupled via a dynamic hologram, Sov. J. Quantum Electron. 17 (1990) 49–52.

[15] L. Solymar, D. Webb, A. Grunnet-Jepsen, The Physics and Application of Photorefractive Materials, Clarendon, Oxford, 1996.

[16] G.C. Valley, G.J. Dunning, Observation of optical chaos in a phase conjugate resonator, Opt. Lett. 9 (1984) 513–515.

[17] A. Yariv, D. Pepper, Amplified reflection, phase conjugation, and oscillation in degenerate four-wave mixing, Opt. Lett. 1 (1977) 16–18.

[18] R.K. Jain, G.J. Dunning, Spatial and temporal properties of a continuous-wave Phase-conjugate resonator based on the photorefractive crystal $BaTiO_3$, Opt. Lett. 7 (1982) 420–422.

[19] B. Sturman, S. Odoulov, M. Goul'kov, Parametric fourwave processes in photorefractive crystals, Phys. Rep. 275 (1996) 197–254.

[20] L.P. Yatsenko, B.W. Shore, K. Bergmann, Theory of a frequency-shifted feedback laser, Opt. Commun. 236 (2004) 183–202.

[21] D. Engin, S. Orlov, M. Segev, G. Valley, A. Yariv, Order disorder phase transition and critical slowing down in photorefractive self-oscillators, Phys. Rev. Lett. 74 (1995) 1743–1746.

[22] A. Shumelyuk, M. Wesner, M. Imlau, S. Odoulov, Double-phase conjugate mirror in nominally undoped $Sn_2P_2S_6$, Opt. Lett. 34 (2009) 734–736.

[23] P. Mathey, Frequency degenerate oscillation in semilinear photorefractive oscillator with reflection grating, Appl. Phys. B 80 (2005) 463–469.

[24] B. Chikh-Bled, M. Aillerie, B. Benyoucef, Dark and photo-conductivity measurement techniques for dielectric materials, applications to $LiNbO_3$, J. Eng. Appl. Phys. 6 (2011) 163–167.

[25] S.M. Kostritskii, M. Aillerie, O.G. Servostyanov, Self-compensation of optical damage in Reduced nominally pure $LiNbO_3$ crystals, J. Appl. Phys. 107 (2010) 123526/1–123526/9.

[26] H. Matsuoka, A. Okamoto, M. Takamura, K. Sato, Reflectivity of photorefractive cross-polarized four-wave mixing in consideration of pump depletion, J. Opt. Soc. Am. B 15 (1998) 1545–1552.

[27] K.R. McDonald, J. Feinberg, Enhanced four-wave mixing by use of frequency shifted waves in photorefractive $BaTiO_3$, Phys. Rev. Lett. 55 (1985) 821–824.

[28] N.C. Kothari, Photorefractive phase-conjugate reflectivity enhancement due to linear absorption, Phys. Rev. A 54 (1996) 5313–5316.

[29] M.G. Reznikov, A.I. Khizhnyak, Properties of a resonator with a wavefront reversing mirror, Sov. J. Quantum Electron. 10 (1979) 533–634.

[30] P.A. Belanger, A. Hardy, A. Siegman, Resonant modes of optical cavities with phase-conjugate mirrors, Appl. Opt. 19 (1980) 602–609.

[31] S. Odoulov, M. Soskin, A. Khyzhnyak, Optical Coherent Oscillators with Degenerate Four-wave Mixing (Dynamic Grating Lasers), Harwood, Chur, London, 1991, pp. 37–39.

[32] A. Novikov, V. Obukhovsky, S. Odoulov, B. Sturman, Explosion instability and coherent optical oscillation in photorefractive crystals, Sov. Phys. JETP 44 (1986) 538–542.

[33] D. Mahgerefteh, J. Feinberg, Explanation of the apparent sublinear photoconductivity of photorefractive barium titanate, Phys. Rev. Lett. 64 (1990) 2195–2198.