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Effect of photoconductivity and oscillation frequency shift on the signal beam intensity in two beam coupling in photorefractive materials

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ABSTRACT

A general theory of the two-beam coupling between a pump beam and a signal beam in photorefractive materials is presented. The coupled wave equations describing the non-linear two beam coupling are derived, based on Maxwell's wave equation. The coupled equation for the intensities of the two beams in the photorefractive crystals with the absorbing properties have been derived analytically. The intensity of the signal beam increases with the increasing crystal thickness, reaches a maximum and then decreases. The influence of energy beam coupling coefficient, oscillation frequency shift, crystal thickness, absorption coefficient and the input beam intensity ratio on the signal beam intensity have been studied in details. The effect of the photoconductivity of the materials on the intensities of the two beams in both the co-directional as well as contra-directional two beam coupling cases have been studied.

1. Introduction

After the discovery of photorefractive effect in the 2nd half of the 20th century, the non-linear optical effects in photorefractive materials have become very challenging and fascinating area of research since the last four decades. Photorefractive materials are amongst the most promising materials and the two beam coupling sometimes referred to as the two-wave mixing in photorefractive materials is a fundamental non-linear optical process responsible for many applications, such as signal processing, optical communications, optical networks, optical computing, real-time holography, image amplification, laser beam steering, optical interconnections, and holographic memory [1–4]. The problem of two-beam coupling in photorefractive materials is completely solvable for certain special cases only under certain assumptions for the nature of the optical interactions [3]. Photorefractive beam coupling is the non-linear interaction of phase and energy between two beams in a photorefractive medium, where transfer of power from one beam to another occurs [4–7]. Through the photorefractive effect the interference pattern of the two beams is transformed into a refractive index grating [8]. The index grating can be considered as a dynamic volume grating, which is both formed by the beams and diffracts them, leading to non-linear beam coupling. This results in transfer of energy and phase between the two interacting beams. The most common theoretical description of the beam coupling in photorefractive materials is known as the coupled wave theory [9–13].

In two-beam coupling, where a transfer of power from one beam to another occurs, different configurations are possible. The configuration in which both the pump and signal beams counter-propagate through the medium is referred to as counter-propagating two-beam coupling [3]. The superposition of the input beams results in a modulated intensity pattern in the medium, as a result of which charge carriers are liberated from the donor atoms and redistribute themselves due to drift or diffusion effects. A charge imbalance ensues resulting in a modulated electric field and thereby, altering the dielectric constant and conductivity of the medium through the electro-optic effect. In general a phase mismatch between the intensity pattern and the electric field modulation may exist, which determines, to a large extent, the coupling (energy transfer) between the two input beams [4]. This energy transfer (beam coupling) between the beams in photorefractive crystals takes place due to a permanent phase mismatch between the refractive index grating and the incident light intensity grating. Maximum energy transfer is obtained when the incident fringe pattern and the photo-induced index change are shifted by $\pi/2$ (in the case of diffusion only) [14].

The problem of two beam coupling in photorefractive materials has being considered by a number of workers. In the present paper we have theoretically analyzed the non-degenerate co-directional and contra-directional two beam coupling inside the photorefractive materials having photoconduction, absorbing and non-absorbing properties under the slowly varying amplitude approximation [8]. In the earlier published literature effects of the

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absorption and input beam intensity ratio on the coupling effect between two beams have not been explored in details. Moreover photoconductivity, oscillation frequency shift and dielectric constant of the materials in two beam coupling have not been considered earlier. In this paper the above effects are considered in details.

2. Theoretical description

Let us consider the interaction of two laser beams of same frequency inside the photorefractive medium. The electric fields $\vec{E}_p$ and $\vec{E}_E$ of the two coupling waves can be written as,

$$\vec{E}_p = \vec{A}_p(z)\exp[j(\omega_p t - \vec{k}_p \cdot \vec{r})] + c \cdot c \tag{1}$$

$$\vec{E}_s = \vec{A}_s(z)\exp[j(\omega_s t - \vec{k}_s \cdot \vec{r})] + c \cdot c \tag{2}$$

where $|\vec{A}_p| = A_p$, $|\vec{A}_s| = A_s$ and ω_p, ω_s are the amplitude and angular frequency of pump beam and signal beam, respectively, and $\vec{k}_p$, $\vec{k}_s$ are the wave vectors of the two waves, $j = \sqrt{-1}$, and t and $\vec{r}$ are the time and space coordinate, respectively.

When the two beam progressing in the medium then due to interference of these two beams interference pattern is formed and the resultant intensity I of the interference pattern is written as,

$$I = |\vec{A}_p|^2 + |\vec{A}_s|^2 + A_p^* A_s e^{j(\Omega t - \vec{k} \cdot \vec{r})} + A_p A_s^* e^{-j(\Omega t - \vec{k} \cdot \vec{r})} \tag{3}$$

In Eq. (3) $\vec{k}$ is the resultant wave vector given by $\vec{k} = \vec{k}_p - \vec{k}_s$ with magnitude $k = 2\pi/\Lambda$, where Λ is the period of the fringe pattern formed due to interference between the two waves and $\Omega = \omega_p - \omega_s$ is the oscillation frequency shift. The resultant intensity I can also be expressed as,

$$I = I_0 + I(r) \tag{4}$$

where I_0 and $I(r)$ are, respectively, the constant and position dependent parts of the resultant intensity and are given by,

$$I_0 = I_p + I_s \equiv |A_p|^2 + |A_s|^2 \tag{6}$$

$$I(r) = A_p^* A_s e^{j(\Omega t - \vec{k} \cdot \vec{r})} + A_p A_s^* e^{-j(\Omega t - \vec{k} \cdot \vec{r})} \tag{7}$$

It can be seen from Eq. (4) that the resultant intensity I contains two terms I_0 and $I(r)$; the first term I_0 is constant and gives the sum of the intensities of the individual waves, whereas the second term $I(r)$ is a spatially varying term with a spatial period Λ. From Eq. (3), it is obvious that within the photorefractive medium the resultant intensity suffers a spatial variation and thereby, generates and redistributes the charge carriers, as a result of which a space charge field is created in the medium. The spatially varying space charge field leads to formation of a refractive-index volume grating, via the Pockel effect [15], which has a spatial phase shift relative to the interference pattern. The spatially changing refractive-index n is given as [16],

$$n = n_0 + \left[\frac{n_1}{2}\exp(j\Phi)\frac{A_p \times A_s}{I_0}\exp\{j(\Omega t - \vec{k} \cdot \vec{r})\} + c \cdot c\right] \tag{8}$$

where $c{\cdot}c$ represents complex conjugation and n_0 is the refractive-index in the absence of light, ϕ is real and is a real and positive number. A scalar grating is considered for the sake of convenience. The phase ϕ indicates the degree to which the index grating is shifted spatially with respect to the light interference pattern [17]. The expression for Φ and Φ related to oscillation frequency shift n_1 can be written as [18,19],

$$\Phi = \Phi_0 + \tan^{-1}(\Omega\tau) \tag{9}$$

$$n_1 = \frac{2\Delta n_s}{\sqrt{1 + \Omega^2\tau^2}} \tag{10}$$

where Φ_0 is a constant phase shift related to the non-local response of the crystal under the interference fringe illumination and τ is the response time of the photorefractive medium and is given by the relation [20],

$$\tau = \frac{\varepsilon}{\sigma_p} \tag{11}$$

where ε is the dielectric constant [21] and σ_p is the photoconductivity of the photorefractive material, respectively. Expression for the term n_1 appearing in Eq. (10) is written as,

$$n_1 = \frac{2\Delta n_s \sigma_p}{\sqrt{\sigma_p^2 + \Omega^2\varepsilon^2}} \tag{12}$$

where Δn_s is the saturation value of the photo-induced index change in the photorefractive materials. The parameters Δn_s and Φ_0 depend not only on the grating spacing Λ and its direction but its material properties also, e.g., the electro-optic coefficients and therefore, on the applied electric field. In photorefractive medium that operates by diffusion only (i.e., no external static electric field) the magnitude of Φ_0 is equal to $\pi/2$ [22,23] (e.g., barium titanate ($BaTiO_3$) crystal) with its sign depending upon the direction of the c-axis.

In order to investigate the coupling between the two light waves solution of the following scalar wave equation is required,

$$\nabla^2 E + \frac{\omega^2}{c^2} n^2 E = 0 \tag{13}$$

where ω is the angular frequency and c is the velocity of light wave and n is the refractive-index of the medium in which wave propagation is considered. Both the beams are assumed to propagate in the x–z plane. The above equation is solved for the steady state so that the amplitudes A_p and A_s are taken to be time independent. The above wave equation is solved under the slowly varying amplitude approximation [9],

$$\left|\frac{d^2 A_p}{dz^2}\right| << \left|\beta_p \frac{dA_p}{dz}\right| \tag{14}$$

$$\left|\frac{d^2 A_s}{dz^2}\right| << \left|\beta_s \frac{dA_s}{dz}\right| \tag{15}$$

where β_p and β_s are, respectively, the z-components of the wave-vectors $\vec{k}_p$ and $\vec{k}_s$ inside the medium. Using the above approximations and substituting the values of E and n from Eqs. (1) and (8), Eq. (13) yields the following relations,

$$2j\beta_p \frac{dA_p}{dz} = \frac{\omega_p^2 n_0 n_1}{c^2 I_0} e^{-j\Phi} A_s^* A_s A_p \tag{16}$$

$$2j\beta_s \frac{dA_s}{dz} = \frac{\omega_s^2 n_0 n_1}{c^2 I_0} e^{j\Phi} A_p^* A_p A_s \tag{17}$$

The energy coupling depends on the relative signs of β_p and β_s and hence, the two-wave coupling phenomenon can be divided into the two categories as (i) co-directional two-wave coupling and (ii) contra-directional two-wave coupling.

2.1. Co-directional two-wave coupling

In case both the beams enter into the medium from the same side the couplings is known as co-directional two wave coupling and in such a case β_p and β_s both are positive. Assuming the two beams entering into the medium symmetrically with respect to the normal (z-axis) to the crystal surface from the left side ($z = 0$), each making an angle θ with the normal one can write,

$$\beta_p = \beta_s = k\cos\theta = \frac{2\pi}{\lambda} n_0 \cos\theta \tag{18}$$

where n_0 is the index of refraction of the medium and $k = 2\pi n_0/\lambda = \Omega/c$. Using Eq. (18), Eqs. (16) and (17) yield,

$$\frac{dA_p}{dz} = -\frac{1}{2I_0}\Gamma|A_s|^2 A_p - \frac{\alpha}{2}A_p \quad (19)$$

$$\frac{dA_s}{dz} = \frac{1}{2I_0}\Gamma^*|A_p|^2 A_s - \frac{\alpha}{2}A_s \quad (20)$$

where the last terms in Eqs. (19) and (20) have been added to account for the material absorption with α as the bulk absorption coefficient of the photorefractive material and Γ as the complex coupling constant given by,

$$\Gamma = j\frac{2\pi n_1}{\lambda \cos\theta}\exp(-j\Phi) \quad (21)$$

where Φ is the angle between the interference pattern and the index gratings. The amplitudes A_p and A_s are in general complex quantities and can be written as,

$$A_p = \sqrt{I_p}\exp(-j\Psi_p) \quad (22)$$

$$A_s = \sqrt{I_s}\exp(-j\Psi_s) \quad (23)$$

where Ψ_p and Ψ_s are the phases of the amplitudes A_p and A_s, respectively. Using Eqs. (8), (22) and (23), Eqs. (19) and (20) gives us,

$$\frac{dI_p}{dz} = -\gamma\frac{I_pI_s}{I_p+I_s} - \alpha I_p \quad (24)$$

$$\frac{dI_s}{dz} = \gamma\frac{I_pI_s}{I_p+I_s} - \alpha I_s \quad (25)$$

$$\frac{d\Psi_p}{dz} = \beta\frac{I_s}{I_p+I_s} \quad (26)$$

$$\frac{d\Psi_s}{dz} = \beta\frac{I_p}{I_p+I_s} \quad (27)$$

where γ and β appearing in Eqs. (24)–(27) are known as intensity and phase coupling constants, respectively, and are related to the complex coupling constant Γ by,

$$\Gamma = \gamma + j\beta \quad (28)$$

From Eqs. (21) and (28) one can write γ and β as,

$$\gamma = \frac{2\pi n_1}{\lambda \cos\theta}\sin\Phi \quad (29)$$

$$\beta = \frac{2\pi n_1}{\lambda \cos\theta}\cos\Phi \quad (30)$$

where λ is the wavelength of the laser beam and θ is the half the angle between the beams inside the photorefractive medium. For photorefractive crystal that operates by diffusion only, $\Phi_0 = \pi/2$. Therefore, using Eqs. (12), (9) and (29), the nonlinear energy coupling constant γ can be written as,

$$\gamma = \frac{\gamma_0}{1+\Omega^2\tau^2} = \frac{\gamma_0\sigma_p^2}{\sigma_p^2+\Omega^2\varepsilon^2} \quad (31)$$

where γ_0 is the coupling constant for the case of degenerate two wave mixing (i.e., $\Omega = \omega_p - \omega_s = 0$) and is given by the expression,

$$\gamma_0 = \frac{4\pi\Delta n_s}{\lambda \cos\theta} \quad (32)$$

The phase coupling constant γ and β depends on Ω, τ and γ_0 through the relation,

$$\beta = \frac{\Omega\varepsilon\gamma_0\sigma_p^2}{2\sigma_p(\sigma_p^2+\Omega^2\varepsilon^2)} \quad (33)$$

Substituting the values of γ and β from Eqs. (31) and (33) into Eqs. (24)–(27) yields the following steady state coupled wave equations as,

$$\frac{dI_p}{dz} = -\left[\frac{\gamma_0\sigma_p^2}{\sigma_p^2+\Omega^2\varepsilon^2}\right]\frac{I_pI_s}{I_p+I_s} - \alpha I_p \quad (34)$$

$$\frac{dI_s}{dz} = \left[\frac{\gamma_0\sigma_p^2}{\sigma_p^2+\Omega^2\varepsilon^2}\right]\frac{I_pI_s}{I_p+I_s} - \alpha I_s \quad (35)$$

$$\frac{d\Psi_p}{dz} = \left[\frac{\Omega\varepsilon\gamma_0\sigma_p^2}{2\sigma_p(\sigma_p^2+\Omega^2\varepsilon^2)}\right]\frac{I_s}{I_p+I_s} \quad (36)$$

$$\frac{d\Psi_s}{dz} = \left[\frac{\Omega\varepsilon\gamma_0\sigma_p^2}{2\sigma_p(\sigma_p^2+\Omega^2\varepsilon^2)}\right]\frac{I_p}{I_p+I_s} \quad (37)$$

Eqs. (34) and (35) are coupled equations for the intensities and Eqs. (36) and (37) are coupled equations for the phases of the two beams. The direction of energy transfer between the two beams is governed by the sign of γ which depends on the orientation of the crystal axes. For positive γ, I_s is an increasing function and I_p is a decreasing function of z for non-absorbing materials ($\alpha = 0$) and under such condition energy is transferred from the pump beam (I_p) to the signal beam (I_s). Eqs. (34) and (35) can be solved to give the expressions for the intensities $I_p(z)$ and $I_s(z)$ as,

Now, adding Eqs. (34) and (35) and integrating with respect to z yields,

$$I_p(z) + I_s(z) = \{I_p(0) + I_s(0)\}\exp(-\alpha z) \quad (38)$$

Using Eq. (38), Eqs. (34) and (36) give the following differential equations,

$$\frac{dI_p}{dz} - \frac{\gamma_0\sigma_p^2}{I_0(\sigma_p^2+\Omega^2\varepsilon^2)}I_p^2\exp(\alpha z) + \left(\frac{\gamma_0\sigma_p^2}{\sigma_p^2+\Omega^2\varepsilon^2} + \alpha\right)I_p = 0 \quad (39)$$

$$\frac{dI_s}{dz} + \frac{\gamma_0\sigma_p^2}{I_0(\sigma_p^2+\Omega^2\varepsilon^2)}I_s^2\exp(\alpha z) - \left(\frac{\gamma_0\sigma_p^2}{\sigma_p^2+\Omega^2\varepsilon^2} - \alpha\right)I_s = 0 \quad (40)$$

where $I_0 = I_p(0) + I_s(0)$. Eqs. (39) and (40) can be integrated to yield the following equations,

$$I_p(z) = I_p(0)\frac{(1+m)\exp(-\alpha z)}{m + \exp(\gamma_0\sigma_p^2 z/(\sigma_p^2+\Omega^2\varepsilon^2))} \quad (41)$$

$$I_s(z) = I_s(0)\frac{(1+m)\exp(-\alpha z)}{1 + m\exp(-(\gamma_0\sigma_p^2 z/(\sigma_p^2+\Omega^2\varepsilon^2)))} \quad (42)$$

where m is the input intensity ratio given by,

$$m = \frac{I_p(0)}{I_s(0)} \quad (43)$$

For extremely small intensity of the signal beam as compared to that of the pump beam [$I_s(0) << I_p(0)$], i.e. $m >> 1$, Eq. (38) can be written as,

$$I_s(z) = I_s(0)\exp\left[\left(\frac{\gamma_0\sigma_p^2}{\sigma_p^2+\Omega^2\varepsilon^2} - \alpha\right)z\right] \quad (44)$$

Thus, it is evident from the above equation that in order to amplify the signal beam one requires the condition ($\alpha < (\gamma_0\sigma_p^2 z/\sigma_p^2 + \Omega^2\varepsilon^2)$). From Eqs. (34) and (35) one can see that in the absence of absorption ($\alpha = 0$), the signal beam gains energy from the pump beam which decays exponentially. In the presence of material absorption ($\alpha > 0$), the signal beam can still be amplified provided the gain due to beam coupling is large enough to overcome the loss. The intensity of the signal beam increases with the increasing crystal thickness reaches a maximum and afterwards decreases exponentially. The crystal thickness (L) corresponding to the intensity maximum of the signal beam is obtained by differentiating Eq.

(35) with respect to z and equating the term $dI_s(z)/dz$ to zero, which yields,

$$L = L_m = \frac{\sigma_p^2 + \Omega^2\varepsilon^2}{\gamma_0\sigma_p^2}\log_e\left\{\frac{m[\sigma_p^2 - \alpha(\sigma_p^2 + \Omega^2\varepsilon^2)]}{\alpha(\sigma_p^2 + \Omega^2\varepsilon^2)}\right\} \tag{45}$$

As pointed out earlier, the direction of energy transfer is governed by the sign of $(\gamma_0\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2))$ and for positive $(\gamma_0\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2))$ the energy is transferred from the pump beam to the signal beam while for negative $(\gamma_0\sigma_p^2/(\sigma_p^2 + \Omega^2\varepsilon^2))$ the pump beam gains energy from the signal beam which is undesirable for the amplification of the weaker (signal) beam. For $m\exp(-\gamma_0\sigma_p^2L/(\sigma_p^2 + \Omega^2\varepsilon^2)) << 1$, $I_s(L)$ becomes $[I_p(0)+I_s(0)]\exp(-\alpha L)$ and under this condition the signal beam takes practically all the energy of the pump beam and decays exponentially afterwards due to material absorption.

2.2. Contra-directional two-beam coupling

When the beams enter into the medium from the opposite sides the coupling is known as contra-directional two-beam coupling. Contrary to the case of the co-directional two-beam coupling, where the sum of the beam intensities is a constant of integration for the non-absorbing medium, in the contra-directional two-beam coupling, the difference of the beam intensities is a constant of integration. In addition, the forms of the coupled equations which govern the wave amplitudes also differ from those of the co-directional coupling case, leading to the qualitative differences in the energy exchanges between the two beams in the two cases. Under such condition Eqs. (16) and (17) yields a set of coupled equations as,

$$\frac{dI_p}{dz} = -\left[\frac{\gamma_0\sigma_p^2}{\sigma_p^2 + \Omega^2\varepsilon^2}\right]\frac{I_pI_s}{I_p + I_s} - \alpha I_p \tag{46}$$

$$\frac{dI_s}{dz} = -\left[\frac{\gamma_0\sigma_p^2}{\sigma_p^2 + \Omega^2\varepsilon^2}\right]\frac{I_pI_s}{I_p + I_s} + \alpha I_s \tag{47}$$

The above two equations can be solved for the non-absorbing medium ($\alpha = 0$) for which the difference of the two beam intensities $[I_s(0)+I_p(0)]$ is a constant and for such a case these equations have the solutions [1],

$$I_p^{\alpha=0}(z) = f(z) - C \tag{48}$$

$$I_s^{\alpha=0}(z) = f(z) + C \tag{49}$$

where C and $f(z)$ are given by,

$$C = \frac{\{I_s(0) + I_p(0)\}}{2} \tag{50}$$

$$f(z) = \sqrt{C^2 + B\exp\left(-\frac{\gamma_0\sigma_p^2 z}{\sigma_p^2 + \Omega^2\varepsilon^2}\right)} \tag{51}$$

With the value of B given by,

$$B = I_s(0)I_p(0) \tag{52}$$

The constants B and C can be more conveniently expressed in terms of initial intensities of the beams 1 and 2, i.e., $I_p(0)$ and $I_s(L)$ as,

$$B = I_p(0)I_s(L)\frac{I_p(0) + I_s(L)}{I_s(L) + I_p(0)\exp(-(\gamma_0\sigma_p^2L/(\sigma_p^2 + \Omega^2\varepsilon^2)))} \tag{53}$$

$$C = \frac{1}{2}\frac{I_s^2(L) - I_p^2\exp(-(\gamma_0\sigma_p^2L/(\sigma_p^2 + \Omega^2\varepsilon^2)))}{I_s(L) + I_p(0)\exp(-(\gamma_0\sigma_p^2L/(\sigma_p^2 + \Omega^2\varepsilon^2)))} \tag{54}$$

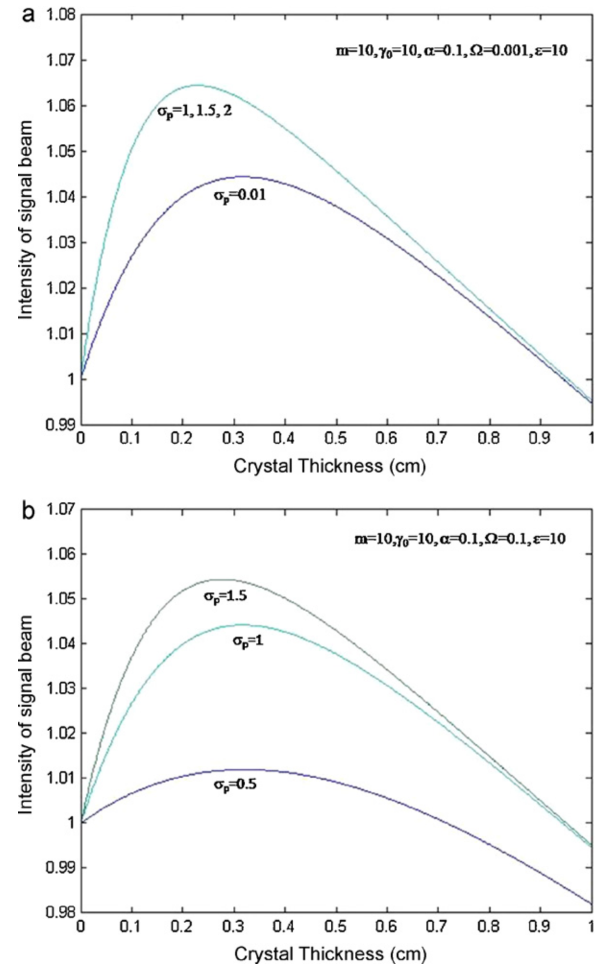

Fig. 1. (a) and (b) Intensity variation of the signal beam with the crystal thickness for constant values of m (=10), γ_0 (=10), α (=0.1), Ω (=0.001), ε (=10) and m (=10), γ_0 (=10), α (=0.1), Ω (=0.1), ε (=10) with different values of σ_p, respectively.

In the case of absorbing medium ($\alpha > 0$) the closed form solutions of Eqs. (46) and (47) are not available in general, however, very good approximate solutions [1,2] are given by,

$$I_p^{\alpha}(z) = I_p^{\alpha=0}(z)\exp(-\alpha z) \tag{55}$$

$$I_s^{\alpha}(z) = I_s^{\alpha=0}(z)\exp[\alpha(z - L)] \tag{56}$$

3. Results and discussion

3.1. Co-directional two-beam coupling

It is clear from Eqs. (41) and (42) that for the co-directional two beam coupling, the intensity of the signal beam is an increasing function of z while that of the pump beam is a decreasing function of z for the positive value of coupling coefficient $(\gamma_0\sigma_p^2L/(\sigma_p^2 + \Omega^2\varepsilon^2))$ in non-absorbing materials. In such a case the signal beam gains energy from the pump beam which decays exponentially. Variation of the intensity of the signal beam with the crystal thickness for constant values of m (=10), γ_0 (=10), α (=0.1), Ω (=0.001), ε (=10) and m (=10), γ_0 (=10), α (=0.1), Ω (=0.1), ε (=10) with different values of σ_p = 0.01, 1, 1.5, and 2 are shown in Fig. 1(a and b), respectively. The intensity of the signal beam increases with the increasing crystal thickness and reaches its maximum value 1.0443 for σ_p = 0.01 and 1.0645 for σ_p = 1, 1.5, 2 then decreases due to material absorption. It can be seen that if the oscillation frequency shift Ω (=0.001) is very lower we cannot differentiate the effect of photoconductivity

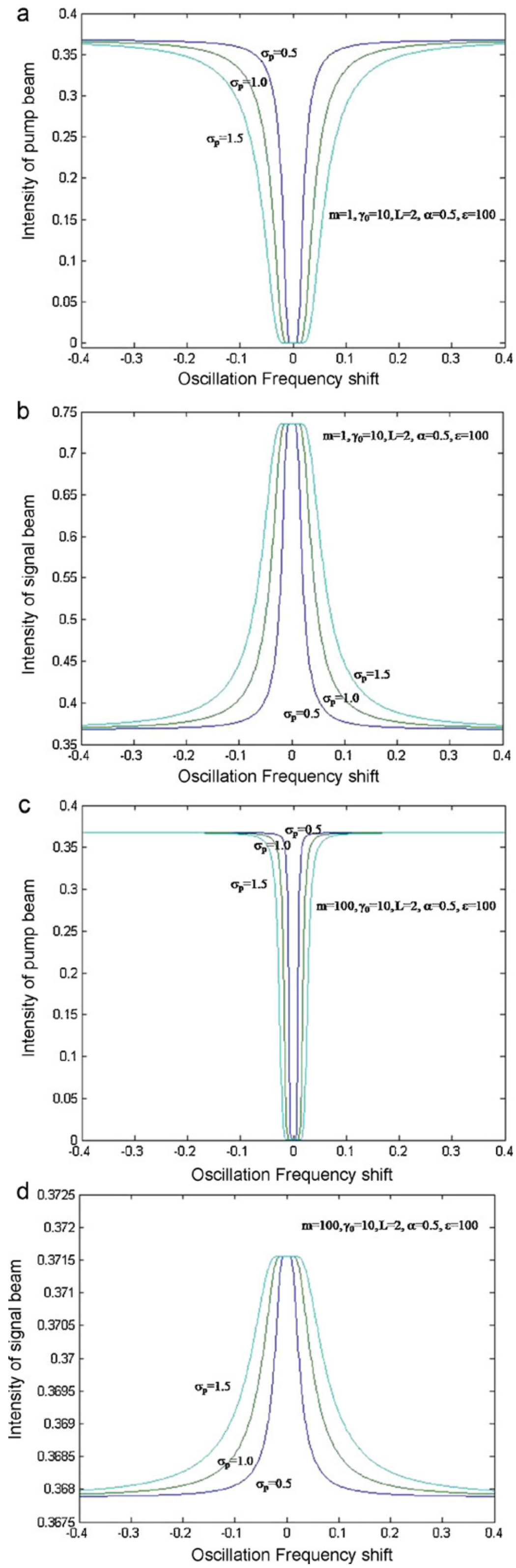

Fig. 2. (a)–(d) The variation of intensity of the pump and signal beam with oscillation frequency shift for constant values of m (=1), γ_0 (=10), α (=0.5), L = 2, ε (=100) (a and b) and m (=1), γ_0 (=10), α (=0.5), L = 2, ε (=100) (c and d) with different values, respectively.

on the intensity of the signal beam, the graphs are coincided for σ_p = 1.0, 1.5 and 2 (Fig. 1a) but if the increment in the oscillation frequency shift is 100 times more than that of the previous value, the amplification of signal beam is effected by the photoconductivity (Fig. 1b). When the oscillation frequency shift increases, the effect of photoconductivity becomes stronger and stronger.

Fig. 2(a and d), respectively, show the variation of intensity of the pump and signal beam with oscillation frequency shift for constant values of m (=1), γ_0 (=10), α (=0.5), L = 2, ε (=100) (Fig. 2a and b) and m (=1), γ_0 (=10), α (=0.5), L = 2, ε (=100) (Fig. 2c and d) with different values of σ_p = 0.5, 1.0 and 1.5, respectively. The intensity of the pump beam decreases with the increase of oscillation frequency shift, reaches its minimum value afterwards increases and finally reaches its initial value again whereas the intensity of the signal beam increases with the increase of oscillation frequency shift, reaches its maximum value afterwards decreases and finally reaches its minimum value again. Both the curves are symmetric about Ω = 0 line. In such a case the signal beam gains energy from the pump beam which is an inherent feature of two beam coupling in photorefractive materials. It is interesting to note here that as we go from lower to the higher value of input intensity ratio (m = 1 to m = 100), the decrement in the intensity peak height ($I_s max$ = 0.7357 for m = 1 and $I_s max$ = 0.3716 for m = 100) of the signal beam is observed. Thus one can say that the coupling between two beans is stronger for lower value of input intensity ratio (Fig. 2b) than that of the higher values (Fig. 2d).

Fig. 3(a and b), respectively, show the variation of the intensity of the pump and signal beam with the photoconductivity of the photorefractive materials for constant values of m (=10), γ_0 (=10), L (=2), Ω (=0.2), ε (=10) with different values of α (=0.2, 0.5, 0.8, and

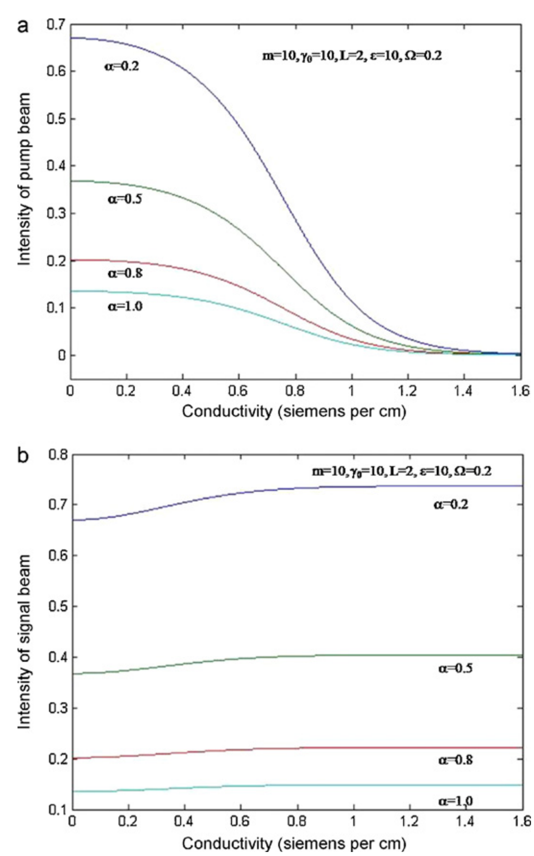

Fig. 3. (a) and (b) The variation of the intensity of the pump and signal beam with the photoconductivity of the photorefractive materials for constant values of m (=10), γ_0 (=10), L (=2), Ω (=0.2), ε (=10) with different values of α, respectively.

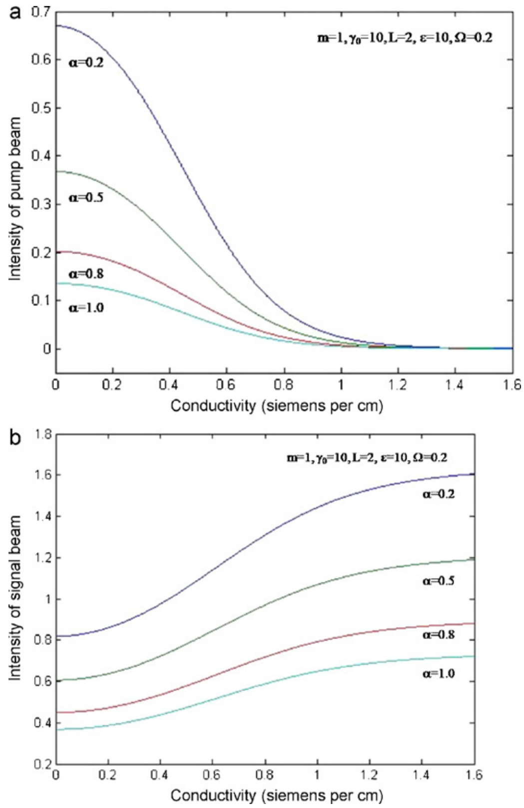

Fig. 4. (a) and (b) Intensity variation of the pump and signal beam with the photoconductivity of the photorefractive materials for constant values of m (=1), γ_0 (=10), L (=2), Ω (=0.2), ε (=10) with different values of α, respectively.

1.0), respectively. It can be seen that the intensity of the pump beam decreases with the increase of the photoconductivity of the materials (Fig. 3a) whereas the intensity of the signal beam increases with the increase of the photoconductivity of the materials and after a certain value it becomes saturate (Fig. 3b).

The variation of the intensity of the pump and signal beam with the photoconductivity of the photorefractive materials for constant values of m (=1), γ_0 (=10), L (=2), Ω (=0.2), ε (=10) with different values of α (=0.2, 0.5, 0.8, and 1.0) are shown in Fig. 4(a and b), respectively. Intensity of the pump beam decreases with the increase of the photoconductivity of the materials (Fig. 4a) whereas the intensity of the signal beam decreases with the increase of the photoconductivity of the materials and after a certain value it becomes saturate (Fig. 4b). Here, it is interesting to note that the transfer of energy from pump beam to the signal beam is much effective in the case when $m = 1$, than that of the case when $m = 10$. Thus one can say that the coupling between the two beams is much strong if the intensity difference is very small.

3.2. Contra-directional two-beam coupling

Variation of the intensity of pump beam and signal beam with crystal thickness for the constant value of m (=100), γ_0 (=10), α (=1.0), ε (=10), Ω (=0.1) with different values of $\sigma_p = 1.0$, $\sigma_p = 1.5$, $\sigma_p = 2.0$ and $\sigma_p = 0.4$ are shown in Fig. 5(a and d), respectively. As expected the pump beam loses its energy whereas the signal beam gains energy in going from one surface of the crystal to the other surface. It can be seen that the intensity of the signal beam increases with increase of crystal thickness and reaches its maximum at certain crystal thickness ($L = 1.486$ for $\sigma_p = 1.0$, $L = 1.6184$ for $\sigma_p = 1.5$ and $L = 1.6763$ for $\sigma_p = 2.0$) then decreases due to the materials absorption. Here it is interesting one that if we select the

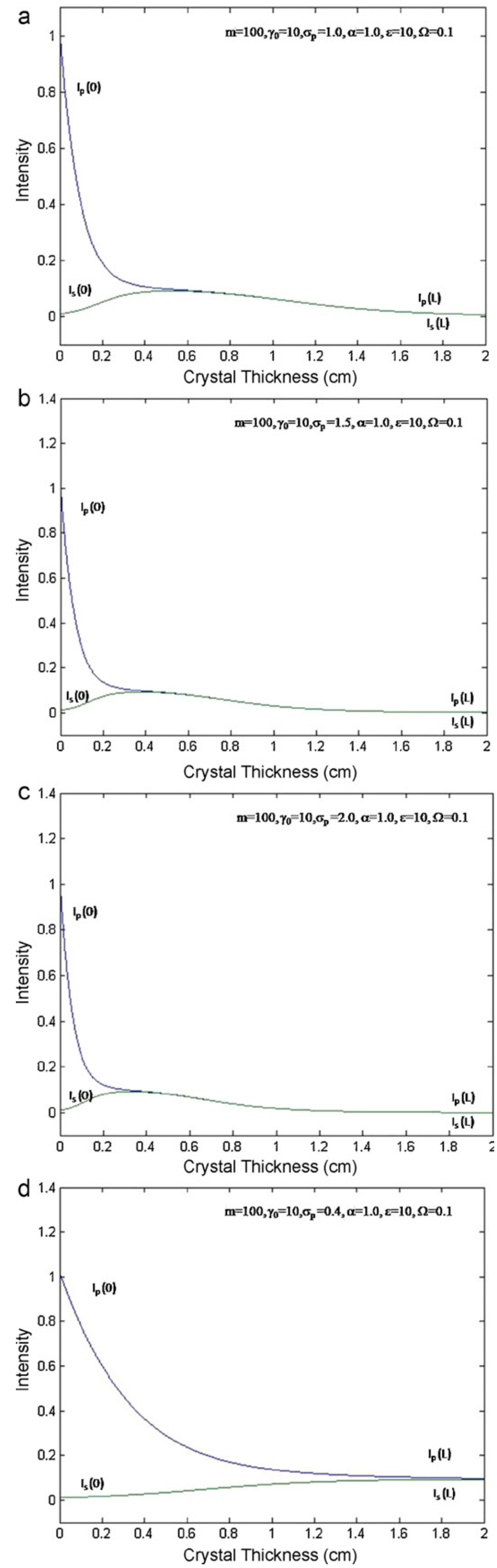

Fig. 5. (a)–(d) Intensity variation of pump beam and signal beam with crystal thickness for the constant value of m (=100), γ_0 (=10), α (=1.0), ε (=10), Ω (=0.1) with different values of σ_p, respectively.

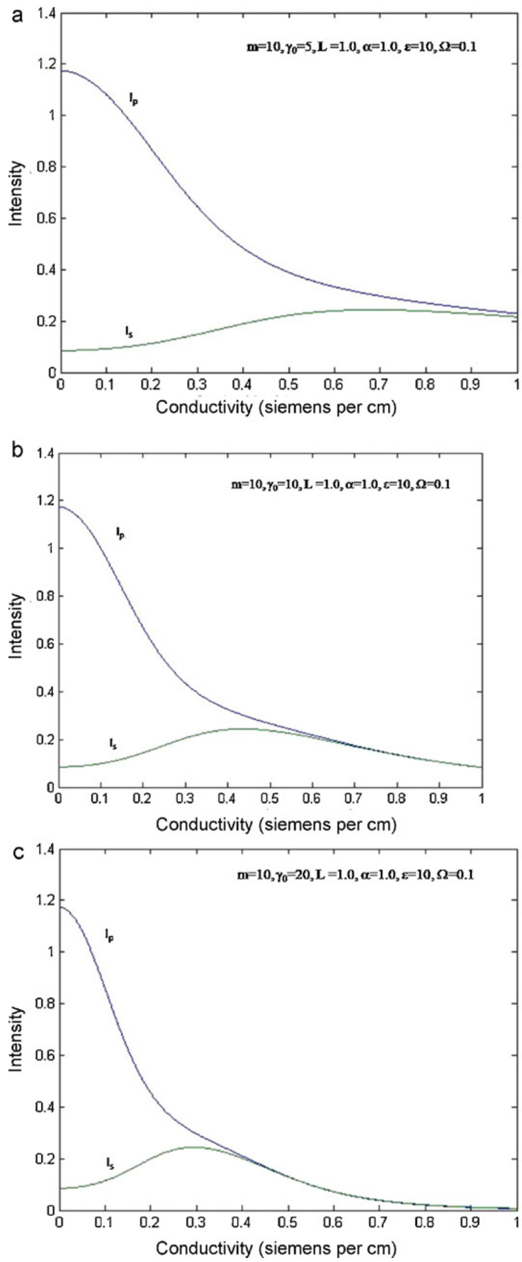

Fig. 6. (a)–(c) Intensity variation of pump and signal beams with photoconductivity of the materials for the constant value of m (=10), L = 1.0, α (=1.0), ε (=10), Ω (=0.1) with different values of γ_0 respectively.

material of higher photoconductivity $\sigma_p \geq 1.0$ the intensity peak height shifted towards the higher value of crystal thickness but there is no change in intensity peak height (I = 0.0913). But if the photoconductivity of the material $\sigma_p \leq 0.4$ (Fig. 5d) the signal beam does not gain energy while pump beam loses its energy, consequently one can say that either there is no coupling between pump and signal beam or absorption of the material is much stronger than that of the coupling effect.

Variation of the intensity of pump and signal beams with photoconductivity of the materials for the constant value of m (=10), L = 1.0, α (=1.0), ε (=10), Ω (=0.1) with different values of $\gamma_0 = 5$, $\gamma_0 = 10$ and $\gamma_0 = 20$ are shown in Fig. 6(a–c), respectively. Intensity of the pump beam decreases whereas signal beam intensity increases with the increase of the photoconductivity of the materials. It can be seen that if we increase the value of γ_0 the intensity peak of the signal beam shifted towards the materials of lower photoconductivity (σ_p = 0.6745 for $\gamma_0 = 5$, σ_p = 0.4335 for $\gamma_0 = 10$ and σ_p = 0.2938 for $\gamma_0 = 20$) but there is no change in intensity peak height of the signal beam (I = 0.2438).

4. Conclusions

We have observed that for small input signals with large input intensity ratio (m » 1), the signal beam amplification requires the condition the coupling coefficient always be greater than the absorption coefficient of the materials. The photoconductivity play an important role in case of non-degenerate two beam coupling but degenerate two beam coupling is unaffected by the photoconductivity of the materials. The intensity of the signal beam increases with the increasing crystal thickness and reaches its maximum value then decreases due to material absorption. If the oscillation frequency shift is very lower nearly equal to zero (degeneracy) wave we cannot differentiate the effect of photoconductivity on the intensity of the signal beam but if the oscillation frequency shift is larger than that of the previous value, the signal beam intensity affected by the photoconductivity of the materials. When the oscillation frequency shift increases, the effect of photoconductivity becomes stronger and stronger. Here it is interesting one that if we select the material of higher photoconductivity $\sigma_p \geq 1.0$ the intensity peak height shifted towards the higher value of crystal thickness but there is no change in intensity peak height (I = 0.0913). But if the photoconductivity of the material $\sigma_p \leq 0.4$ (Fig. 5d) the signal beam does not gain energy while pump beam loses its energy consequently one can say that either there is no coupling between pump and signal beam or absorption of the material is much stronger than that of the coupling effect. If we increase the value of γ_0 the intensity peak of the signal beam shifted towards the materials of lower photoconductivity but there is no change in intensity peak height of the signal beam.

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