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Solar Mass & Dark Energy Dependence Characteristics Study of Black Holes and their role in Galaxy formation and Cosmic Evolution

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Abstract:
In this research paper, we investigate the formation of black holes in the early universe and their fundamental role in galaxy formation and cosmic evolution. By examining key theoretical frameworks such as the Friedmann equations, Jeans instability, the Chandrasekhar limit, Bekenstein-Hawking entropy, and Hawking radiation, we aim to understand how black holes emerged from primordial conditions. These mathematical tools allow us to describe the universe's expansion, the collapse of gas clouds into dense objects, and the critical mass at which objects become black holes. Our study of Hawking radiation reveals that a black hole's lifetime increases with its mass, suggesting that super-massive black holes live longer than their smaller counterparts. This research enhances our understanding of the early universe and provides insights into galaxy formation and black hole evolution over time. It is found that as Ω_Λ increases, the expansion of the universe at late times becomes more pronounced. The universe with a high $\Omega_\Lambda = 0.9$ expands exponentially, showing the dominance of dark energy over gravitational forces from matter. For smaller values of Ω_Λ, such as 0.2 or 0.5, the universe expands more slowly in the past and continues to grow steadily but not as rapidly. It also observed that for black holes with smaller masses (e.g., stellar-mass black holes), the rate of mass loss is relatively high. The graph sharply declines as mass decreases, indicating that small black holes (e.g., those with less than a few solar masses) would evaporate quickly compared to their more massive counterparts.

Keywords: Black hole, Galaxy formation, Hawking radiation, Chandrasekhar limit, Cosmos, Universe.

1. INTRODUCTION

Black Holeshave existed since the beginning of the universe. Black Holes are so massive that not even light can escape their gravity they consume every planet and star in their way due to their high gravitational pull.Over the past several decades, these celestial entities have moved from theoretical constructs to underlying elements in our understanding of cosmic evolution. In modern astrophysics, black holes are not merely exotic remnants of stellar evolution; they play a significant role in shaping the universe's large-scale structure, particularly in the formation of galaxies and the evolution of cosmic matter. Black holes that formed in the early universe, especially those categorized as primordial and supermassive black holes (SMBHs), are of great interest because of their potential influence on the formation of galaxies, star formation, and the overall evolution of the universe(Begelman, et al., 2006).The early universe, shortly after the Big Bang, was a hot and dense plasma dominated by fundamental particles. It was during this period that the seeds for black holes may have been sown. Primordial black holes (PBHs) are theorized to have formed due to quantum fluctuations in the high-energy density of the early universe, possibly within the first few seconds after the Big Bang(Carr & Kuhnel, 2020). These quantum fluctuations could have created regions dense enough to collapse under their own gravity, forming black holes of various sizes. Unlike stellar black holes, which form from the collapse of massive stars, PBHs could range from very small to supermassive sizes, with masses spanning from less than a kilogram to several million times that of the Sun. It's a broad subject of interest for those who are keenly interested to know about space and cosmology(Volonteri, 2012).

The significance of PBHs lies not only in their potential to be the dark matter that constitutes a significant portion of the universe's mass, but also in their contribution to the formation of larger cosmic structures. The formation of SMBHs, typically found at the centers of most galaxies, remains an open question in astrophysics(Reines & Comastri, 2016). It is unclear whether SMBHs formed through the rapid accretion of material onto smaller black holes or if they originated from massive PBHs(Shankar F. et al., 2020). Recently observations of quasars, which are extremely luminous objects powered by SMBHs, at redshifts as high as $z \approx 7$(Banados E. et al., 2018), suggest that some of these black holes were already present within the first billion years after the Big Bang(Kormendy & Ho, 2013).This has profound implications for our understanding of how the universe's first structures coalesced and evolved.The role of black holes, especially SMBHs, in galaxy formation is a subject of intense research(Silk & Rees, 1998). It is now well established that SMBHs reside at the cores of most galaxies, including our own Milky Way. The SMBHs and galaxies both are co-evolved, with the growth of the black hole influencing the formation and evolution of the galaxy, and vice versa(Loeb & Furlanetto, 2013). Several mechanisms are thought to govern this co-evolution, including feedback processes from the black hole itself(Hawking, 1974). As material falls into an SMBH, enormous amounts of energy are released in the form of radiation and jets, which can expel gas and dust from the galaxy's core(Peebles & Nusser, 2010). This feedback can regulate star formation, as it either stimulates or suppresses the collapse of gas clouds into stars.In terms of cosmic evolution, black holes may have influenced the large-scale structure of the universe by acting as seeds around which matter could accumulate, facilitating the formation of galaxies and galaxy clusters (Bromm & Yoshida, 2011). In particular, SMBHs in the centers of galaxies could have helped regulate the distribution of matter and the formation of galaxies through their gravitational pull and feedback processes (Madau & Rees, 2001).Our observations not only confirm predictions made by Einstein's general theory of relativity but also offer insight into the population and characteristics of black holes in the universe (Madau & Rees, 2001). The study of black holes, particularly those formed in the early universe, is crucial for a comprehensive understanding of how the cosmos evolved from a hot, dense state to the vast, structured universe we observe today (Greene, et al., 2020). By investigating the formation and growth of black holes, scientists can be able to answer, key questions regarding the distribution of matter, the role of dark matter and dark energy, and the processes that led to the formation of galaxies (Carr & Kuhnel, 2020).

2. THEORETICAL DESCRIPTION

The formation of black holes in the early universe and their role in galaxy formation and cosmic evoluti on is a profound subject grounded in several fundamental principles of cosmology and astrophysics. This research integrates various theoretical frameworks to investigate the conditions under which black holes form, how they grow, and how they influence the evolution of galaxies and the large-scale structure of the universe.

2.1. Derivation for Friedman Equations and Cosmic Expansion

The Friedman equations, derived from Einstein's General Theory of Relativity, describe the expansion of the universe. These equations provide a theoretical foundation for understanding how the universe evolved from a hot, dense state after the Big Bang to its current state of accelerated expansion(Friedmann, 1924). In the context of black hole formation, the Friedman equations help us model the overall dynamics of the universe, including how matter and energy densities evolved in different eras(Friedmann, 1924). This is essential for determining when and where the conditions for black hole formation could have occurred, particularly for primordial black holes (PBHs), which may have formed due to high-density fluctuations in the early universe(Peebles, 1993).

The Friedmann equations are derived from Einstein's field equations of general relativity, which relate the geometry of spacetime to the energy content of the universe. The metric describing a homogeneous, isotropic universe is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -c^2dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

(1)

Where, $a(t)$ is the scale factor, k is the curvature parameter (k = 0for flat, k = 1 for closed, and k = -1 for open universes),$d\Omega^2$and is the angular component of the metric.

Einstein's field equations are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

(2)

Where, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar,$g_{\mu\nu}$ is the metric, Λ is the cosmological constant, and $T_{\mu\nu}$ is the stress-energy tensor.

For a perfect fluid, the stress-energy tensor is:

$$T_{\mu\nu} = (\rho + \mathrm{p}/c^2)u_\mu u_\nu + \mathrm{p}g_{\mu\nu}$$

(3)

Inserting the FLRW metric and stress-energy tensor into Einstein's field equations and solving the 00-component (time-time) results in the first Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

(4)

To derive the second Friedmann equation, we take the time derivative of the first and use the conservation of energy:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{\mathrm{p}}{c^2}\right) = 0$$

(5)

This leads to the acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3\mathrm{p}}{c^2}\right) + \frac{\Lambda}{3}$$

(6)

2.2. Derivation of the Jeans Instability Equation

Jeans instability is a critical concept in understanding how initial fluctuations in the density of cosmic matter can lead to the formation of large-scale structures, including black holes(Jeans, 1902). The Jeans criterion determines whether a gas cloud will collapse under its own gravity or continue to expand with the universe. When the gravitational forces within a region of space exceed the pressure forces, the region becomes unstable, leading to the formation of dense objects, such as stars or black holes(Jeans, 1902).

Jeans instability describes the conditions under which a region of gas in a homogeneous medium will collapse under its own gravity, leading to the formation of stars or larger structures(Jeans, 1902). This instability arises from a balance between gravitational forces and pressure gradients.

Starting with the linearized equations of hydrodynamics:

1. Continuity equation (mass conservation):

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho v)=0$$

(7)

2. Euler's equation (momentum conservation):

$$\frac{\partial v}{\partial t}+(v\cdot\nabla)v=-\frac{1}{\rho}\nabla P-\nabla\Phi$$

(8)

3. Poisson's equation (relating gravitational potential Φ to density):

$$\nabla^2\Phi=4\pi G\rho$$

(9)

Now, assume small perturbations in density ρ, velocity v andpressure P around an equilibrium state. Let:

-$\rho=\rho_0+\delta\rho$ (10)

- $v=\delta v$ (11)

- $P=P_0+\delta P$ (12)

Linearizing the equations for small perturbations, we obtain:

1. From the continuity equation:

$$\frac{\partial\delta\rho}{\partial t}+\rho_0\nabla\cdot(\delta v)=0$$

(13)

2. From the Euler equation:

$$\frac{\partial\delta v}{\partial t}=-\frac{1}{\rho_0}\nabla\delta P-\nabla\delta\Phi(14)$$

3. From the Poisson equation:

$$\nabla^2\delta\Phi=4\pi G\delta\rho$$

(15)

Next, assume perturbations are plane waves, with $\propto e^{i(k\cdot x-\omega t)}$, where $k$is the wavevector and ω is the frequency. Substituting this into the linearized equations leads to the dispersion relation:

$$\omega^2=c_s^2k^2-4\pi G\rho_0$$

(16)

Where c_s is the speed of sound. For instability ($\omega^2<0$), the condition is:

$$k^2<\frac{4\pi G\rho_0}{c_s^2}$$

(17)

Thus, the wavelength $\lambda = \frac{2\pi}{k}$ must satisfy:

$$\lambda > \lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho_0}}$$

(18)

Where λ_J is the **Jeans length**, below which pressure dominates and above which gravity dominates, leading to collapse.(Binney & Tremaine, 2008)

2.3. Derivation of the Chandrashekhar Limit and Stellar Collapse

The Chandrasekhar limit, which defines the maximum mass a white dwarf star can have before it collapses into a neutron star or black hole, is another key theoretical tool in understanding black hole formation. For non-rotating stars, this limit is approximately 1.4 times the mass of the Sun. If a star's mass exceeds this limit, electron degeneracy pressure can no longer support the star, leading to gravitational collapse and potentially forming a stellar black hole(Chandrasekhar, 1931).

The Chandrasekhar limit is particularly relevant for understanding how black holes formed from the remnants of the first stars, known as Population III stars(Chandrasekhar, 1931). These stars were massive, short-lived, and their remnants could easily collapse into black holes, contributing to the population of early black holes. The collapse of such massive stars also plays a role in explaining the formation of supermassive black holes (SMBHs) observed in the early universe.

It arises from the balance between two opposing forces: **electron degeneracy pressure** (which resists collapse) and **gravitational pressure** (which pulls matter inward). When the star's mass exceeds this limit, gravity overwhelms the degeneracy pressure, causing collapse(Kippenhahn & Weigert, 1990).

Electron Degeneracy Pressure:Degenerate matter follows the principles of quantum mechanics, specifically the Pauli exclusion principle, which states that no two fermions (like electrons) can occupy the same quantum state. At very high densities, electrons are forced into higher-energy states, creating pressure independent of temperature, known as electron degeneracy pressure.(Kippenhahn & Weigert, 1990)

For non-relativistic electrons, the degeneracy pressure is:

$$P_{deg} \propto \left(\frac{N}{V}\right)^{5/3}$$

(19)

For relativistic electrons (relevant for white dwarfs near the Chandrasekhar limit):

$$P_{deg} \propto \left(\frac{N}{V}\right)^{4/3}$$

(20)

Gravitational Pressure:For a star of mass M and radius R, the gravitational pressure (using Newtonian gravity) is:

$$P_{grav} \propto \frac{GM^2}{R^4}$$

(21)

Balancing the Two Forces: In a stable white dwarf, electron degeneracy pressure balances gravitational pressure. Equating the degeneracy and gravitational pressures for a relativistic Fermi gas, we find the mass M_{ch} above which degeneracy pressure cannot prevent collapse:

$$M_{ch} \propto \left(\frac{hc}{G}\right)^{3/2} \frac{1}{\mu_e^2}$$

(22)

The constant of proportionality depends on the mean molecular weight per electron, μ_e, where $\mu_e \approx 2$ for typical white dwarfs (composed of helium, carbon, and oxygen). Chandrasekhar calculated the value to be approximately 1.4 M⊙ (solar masses).

$$M_{ch} = \omega^3 \mu_e^{-2} \left(\frac{hc}{G}\right)^{3/2}$$

(23)

This is the **Chandrasekhar limit**, above which a white dwarf will collapse into a neutron star or black hole(Chandrasekhar, 1931).

2.4. Derivation of Bekenstein-Hawking Entropy and Hawking Radiation

The Bekenstein-Hawking entropy relates the entropy of a black hole to the area of its event horizon. This concept combining thermodynamics, quantum mechanics, and general relativity, links the thermodynamic properties of black holes with their geometry(Bekenstein, 1973)(Hawking, 1975).

Bekenstein-Hawking Entropy Formula:

The entropy S of a black hole is given by the Bekenstein-Hawking entropy formula:

$$S = \frac{k_B A}{4 l_p^2}$$

(24)

Where:S is the entropy of the black hole, A is the area of the black hole's event horizon,k_B is the Boltzmann constant,$l_p = \sqrt{\frac{\hbar G}{c^3}}$is the Planck length, which is a fundamental length scale combining quantum mechanics, general relativity, and the speed of light.

The event horizon area A for a Schwarzschild black hole is related to the black hole's mass M by:

$$A = 16\pi \left(\frac{GM}{c^2}\right)^2$$

(25)

Substituting the expression for A into the entropy equation:

$$S = \frac{k_B 16\pi G^2 M^2}{4 l_p^2 c^4}$$

(26)

Simplifying further using the Planck length$l_p = \sqrt{\frac{\hbar G}{c^3}}$, we get:

$$S = \frac{k_B A}{4 l_p^2} = \frac{4\pi k_B G M^2}{\hbar c}$$

(27)

This shows that the entropy of a black hole is proportional to the square of its mass(Bekenstein, 1973).

Hawking Radiation:-

Hawking showed that black holes emit radiation due to quantum mechanical effects near the event horizon, leading to the **Hawking temperature**:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

(28)

The radiation emitted by the black hole reduces its mass, leading to the eventual evaporation of the black hole. We say as the black hole loses mass, its temperature increases, accelerating the evaporation process(Hawking, 1975).

2.5. Derivation of Cosmic Inflation and Density Perturbations

Cosmic Inflation is a theoretical model that explains the rapid exponential expansion of the universe in the first fraction of a second after the Big Bang. The inflationary phase solves key cosmological puzzles such as the horizon problem and the flatness problem(Guth, 1981). The inflationary model also provides the framework for understanding the origin of density perturbations that lead to the formation of structures like galaxies and clusters(Guth, 1981).

1. Inflationary Expansion:

During inflation, the universe expands exponentially:

$$a(t) \propto e^{Ht}$$

(29)

where:

$a(t)$is the scale factor, describing the size of the universe at a time t and H is the Hubble parameter during inflation, assumed to be nearly constant.

This rapid expansion drives the universe towards a flat geometry and ensures uniformity across vast distances.

2. Quantum Fluctuations During Inflation:

Quantum fluctuations in the inflation field, responsible for driving inflation, give rise to density perturbations. These fluctuations get stretched to cosmic scales during inflation(Mukhanov & Chibisov, 1981).

The comoving curvature perturbation R, which measures these fluctuations, remains nearly constant on super-horizon scales. The power spectrum of the curvature perturbations, $P_R(k)$, is expressed as:

$$P_R(k) \propto A_s \left(\frac{k}{k_*}\right)^{n_s-1}$$

(30)

where:

$A_s$is the amplitude of scalar perturbations, k is the comoving wave number,k_* is a reference wave number (usually taken at the pivot scale) and n_s is the scalar spectral index.

For scale-invariant perturbations,$n_s = 1$. However, observational data suggest $n_s \approx 0.965$, indicating a slight tilt in the power spectrum.

3. Density Perturbations and Structure Formation:

After inflation ends, these curvature perturbations evolve into density perturbations:

$$\frac{\delta\rho}{\rho} = R$$

(31)

These density perturbations are the seeds for the formation of large-scale structures, with regions of higher density collapsing to form galaxies and galaxy clusters (Guth, 1981)(Mukhanov & Chibisov, 1981).

2.6. Derivation for Mass Loss Due to Hawking Radiation

The mass loss of a black hole due to Hawking radiation can be derived from the blackbody radiation emitted by the black hole(Hawking, 1975). Hawking radiation is thermal in nature and is emitted at a temperature T_H, given by:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

(32)

Where: M is the mass of the black hole, $G$is the gravitational constant, c is the speed of light, k_B is the Boltzmann constant, and ħ is the reduced Planck constant.

The rate at which energy (mass) is radiated by a black hole can be estimated using Stefan-Boltzmann law, which states that the power radiated by a black body is proportional to its surface area Ă and temperature $T_H$raised to the fourth power:

$$P = \sigma \breve{A} T_H^4$$

(33)

For a black hole, the surface area of the event horizon is $\breve{A} = 16\pi G^2 M^2 / c^4$, and substituting $T_H$gives the power emitted:

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$$

(34)

This power represents the rate at which the black hole is losing mass (since $P = -dE/dt = -dMc^2/dt$). Dividing the power by c^2 yields the rate of mass loss:

$$\frac{dM}{dt} = -\frac{\hbar c^4}{15360\pi G^2 M^2}$$

(35)

This equation shows that the mass of a black hole decreases over time as a result of Hawking radiation. The rate of mass loss increases as the black hole becomes smaller, eventually leading to the complete evaporation of the black hole over time(Hawking, 1975).

2.7. Derivation for Black Hole Lifetime Due to Hawking Radiation

The lifetime τ of a black hole due to Hawking radiation can be derived from the rate of mass loss(Hawking, 1975). The temperature T_H of a black hole is given by:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

(36)

Where: M is the mass of the black hole, $G$is the gravitational constant, c is the speed of light, k_B is the Boltzmann constant, and ħ is the reduced Planck constant.

The power radiated by a black hole can be determined from the Stefan-Boltzmann law, which states that the power P emitted by a black body is proportional to the fourth power of its temperature:

$$P = \sigma \breve{A} T_H^4$$

(37)

For a black hole, the surface area Ă of the event horizon is given by $\breve{A} = 16\pi G^2 M^2 / c^4$. Thus, the power radiated becomes:

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$$

(38)

The energy loss is related to the mass loss by the equation $E = Mc^2$. Therefore, the rate of mass loss is:

$$\frac{dM}{dt} = -\frac{P}{c^2} = -\frac{\hbar c^4}{15360\pi G^2 M^2}$$

(39)

To find the lifetime τ of the black hole, we set up the differential equation:

$$\tau = -\frac{M}{\frac{dM}{dt}} = \frac{15360\pi G^2 M^3}{\hbar c^4}$$

(40)

This indicates that the lifetime of a black hole is proportional to the cube of its mass:
$\tau \propto M^3$
(41)
The implications are significant; smaller black holes evaporate much more quickly than larger ones, leading to the conclusion that supermassive black holes can exist for exceedingly long timescales(Hawking, 1975).

3. RESULT AND DISSCUSSION

Evolution of the Scale factor a(t) describes the expansion of universe as a function of timet :

From my studies of plot all the curves start near zero at early times (representing the early universe) and grow over time. As time increases, the scale factor $a(t)$ also increases, representing the expansion of the universe.The initial part of the curve rises slowly due to the dominance of radiation and matter in the early universe. At later times, when dark energy becomes dominant, the curves rise more steeply.
Effect of Matter Density: A higher matter density results in a slower expansion at early times because matter slows down the rate of expansion due to gravitational attraction.Universes with higher show a more gradual increase in the scale factor during the matter-dominated era. In contrast, universes with lower Ω_m(e.g., when dark energy dominates) show faster expansion earlier on.
Effect of Dark Energy: Curves with higher $\Omega_\Lambda$show a more rapid expansion at later times. Dark energy has a repulsive effect, accelerating the expansion of the universe, especially at more recent cosmic times. As Ω_Λ increases, the universe transitions earlier from deceleration (due to matter) to acceleration (due to dark energy). Curves with larger $\Omega_\Lambda$values show sharper upward slopes at later times.For example, when$\Omega_\Lambda = 0.9$, the expansion rate significantly accelerates at around 6–7 Gyr, resulting in an exponential rise of the scale factor.
Effect of Curvature: If $\Omega_k > 0$(open universe), the universe expands faster at later times because of the lower gravitational pull of matter and dark energy. The scale factor increases more rapidly as the universe becomes more curved.
Fromthe plot as Ω_Λ increases (from 0.2 to 0.9 in the example), the expansion of the universe at late times becomes more pronounced. The universe with a high Ω_Λ = 0.9 expands exponentially, showing the dominance of dark energy over gravitational forces from matter. For smaller values of Ω_Λ, such as 0.2 or 0.5, the universe expands more slowly in the past and continues to grow steadily but not as rapidly.
In summary, as $\Omega_\Lambda$increases, the universe expands faster and may approach an exponential rate of growth.

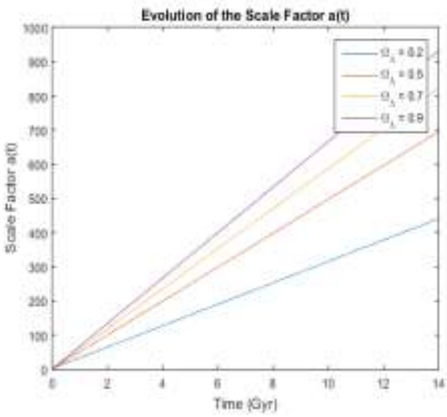

Fig. i) Scale factor a(t) vs. time curve

Jeans length as a function of temperature for different Densities shows the formation of celestial entities:

In the fig.(ii) the curves show that the Jeans length increases with temperature for all densities. The relationship between Jeans length and temperature is approximately proportional to the square root of temperature,$\lambda_J \propto \sqrt{T}$, resulting in curves that steadily rise as temperature increases.

From the plot at lower gas densities (e.g., $\rho_0 = 10^{-18} kg/m^3$), the Jeans length is larger across all temperatures, indicating that in a low-density medium, the critical size for collapse is larger. This happens because a lower density reduces the gravitational pull, making it harder for the gas to collapse.Conversely, at higher gas densities (e.g., $\rho_0 = 10^{-16} kg/m^3$), the Jeans length is smaller for the same temperature, indicating that the gas cloud will collapse more easily at smaller sizes. In a denser medium, gravity is stronger, favoring collapse at smaller scales.

The curves show that the Jeans length decreases with increasing density, following the inverse square root relationship $\lambda_J \propto 1/\sqrt{\rho_0}$. This results in steeply declining curves where the Jeans length rapidly shrinks as density increases.For higher temperatures (e.g., 1000K), the Jeans length is larger for all densities, meaning that at a higher temperature, a gas cloud needs to be much larger to collapse, as thermal pressure opposes gravity more effectively.For lower temperatures (e.g., 10K), the Jeans length is significantly smaller, indicating that in cold environments, even small gas clouds can collapse due to weaker thermal pressure support leads to the formation.

In summary from the plot we can say that Jeans length is a critical scale that determines whether a gas cloud will collapse under gravity or remain stable due to thermal pressure. These insights help understand where and when star formation and other gravitational instabilities can occur in various astrophysical environments.

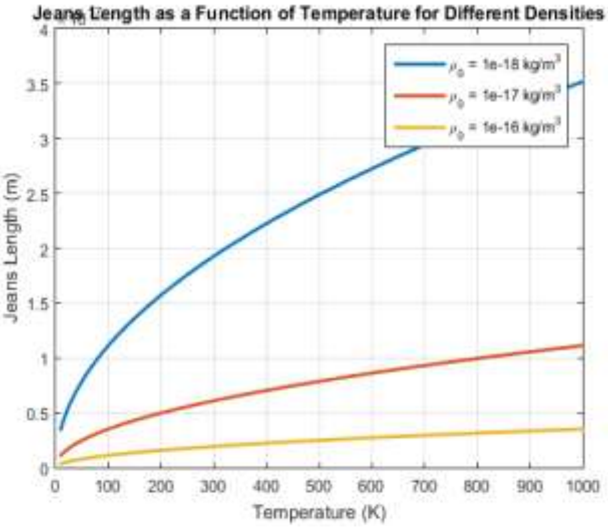

Fig. ii) Jeans length as a function of temperature for different Densities Curve

Journey from dying star to the Black Hole, Chandrashekhar limit (solar masses) as a function of mean molecular weight per electron (μ_e) :

Our plot shows a decreasing trend as the mean molecular weight per electron increases. This indicates that as the mean molecular weight per electron rises, the Chandrasekhar limit (measured in solar masses) decreases. This means that a white dwarf can support less mass before collapsing. Although the slopes of the curves can vary depending on the specific values chosen, all curves consistently illustrate a decline, reinforcing the inverse relationship between the mean molecular weight per electron and the Chandrasekhar limit.

Values of μ_e:

- μ_e= 1.8: This indicates a lighter composition (mainly hydrogen and helium), allowing white dwarfs to support more mass before collapse, as shown by the highest Chandrasekhar limit. Consequently, these stars can be larger and may delay their evolutionary processes.
- μ_e= 2: This value is representative of a standard composition, often seen in carbon-oxygen white dwarfs. The curve for this value indicates a moderate Chandrasekhar limit, reflecting a balance between the gravitational force and electron degeneracy pressure.
- μ_e= 2.5: This value indicates a heavier composition, likely including oxygen and neon. The μ sub e curve shows a lower Chandrasekhar limit, meaning these stars collapse at a smaller mass, resulting in faster formation of neutron stars or black holes.

The plot shows an inverse relationship that highlights the importance of elemental composition in a white dwarf's fate. As the mean molecular weight per electron increases, electron degeneracy pressure's ability to counteract gravitational collapse decreases, leading to a lower maximum mass known as the Chandrasekhar limit.

The plotted curves illustrate how mean molecular weight relates to the Chandrasekhar limit, showing that variations in elemental composition significantly affect a white dwarf's stability and fate.

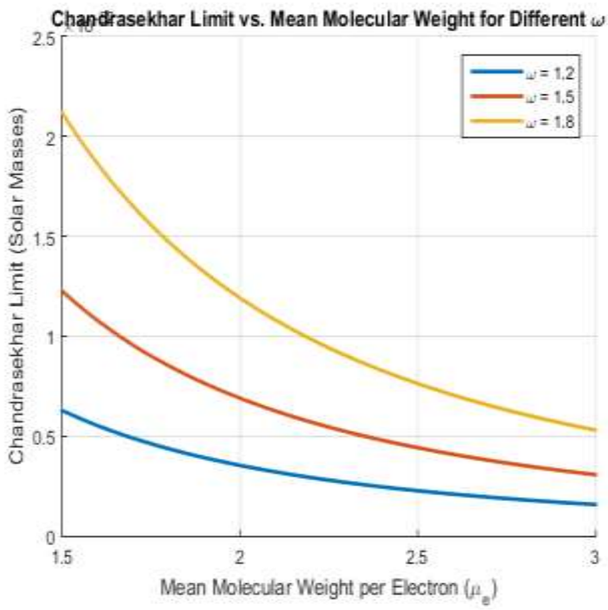

Fig. iii) Chandrashekhar limit vs. Mean molecular weight per electron Curve

Role of Black Holes in the Evolution of the Universeor TheBlack Hole Entropy S as a function of the Event Horizon area A using the Bekenstein-Hawking entropy formula :

The curve illustrates a clear linear increase in entropy as the area of the event horizon increases. This relationship exists because entropy is directly proportional to the area of the black hole's event horizon. The relevant equation indicates that for every increase in the event horizon area, the entropy increases correspondingly, which is represented as a straight line on the plot.

From the plot, it is evident that as the event horizon area grows—indicating a more massive black hole—the entropy increases significantly. The entropy of a black hole reflects the amount of "information" it contains, suggesting that larger black holes possess more information or degrees of freedom. For example, when the area increases by a factor of ten, the entropy also increases by a factor of ten, maintaining a consistent linear relationship.

The plot steepens at larger event horizon areas (A), indicating that as these areas grow—like in supermassive black holes—entropy increases significantly. Small mass increases for such black holes result in substantial entropy rises due to the quadratic relationship between mass and event horizon area. In contrast, black holes with smaller event horizon areas (under a few million square meters) have relatively low entropy, storing less information than their larger counterparts. This supports the idea that the number of microscopic states within a black hole increases with its size.

In summary, our plot highlights the linear relationship between black hole entropy and the event horizon area. Larger black holes, with more massive event horizons, contain significantly more entropy, and this trend continues to scale up as the area increases. The consistent linearity across different scales reinforces the foundational role of the Bekenstein-Hawking entropy formula in describing the internal structure and behaviour of black holes.

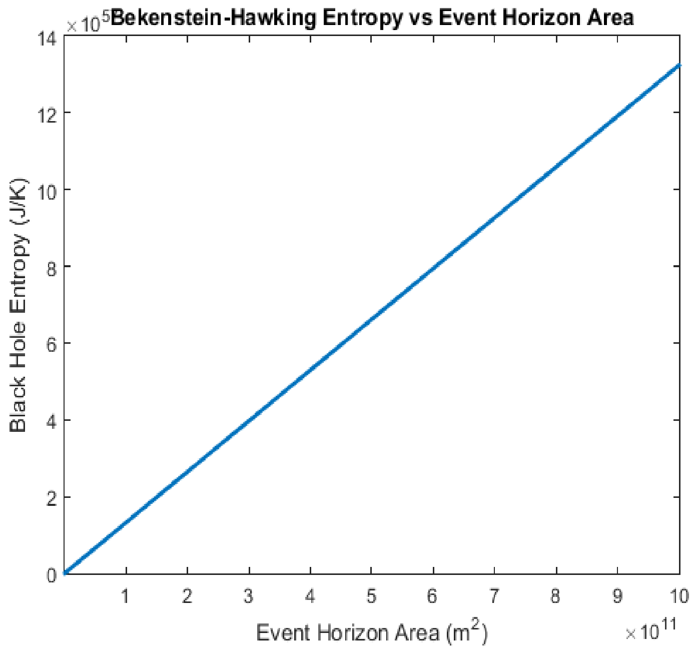

Fig. iv) Black Hole Entropy (J/K) vs. Event horizon Area (m^2) Curve

Structure Formation and the Inflationary model of the early Universe or The Power Spectrum of Density Perturbations$P_R(k)$ as a function of the Comoving wavenumber k, for different values of the Scalar spectral index n_s:

Our plot uses a log-log scale, showing the relationship between the power spectrum $P_R(k)$on the vertical axis and the comoving wavenumber k on the horizontal axis. The log scale helps highlight variations across many orders of magnitude in both k and$P_R(k)$.

The curve exhibits a red-tilted spectrum, meaning the power decreases as k increases (smaller spatial scales). This implies that there is more power on larger scales (small k, corresponding to larger structures in the universe) compared to smaller scales (large k, corresponding to smaller structures).The slope of the spectrum is determined by the scalar spectral indexn_s. The curve becomes less steep with higher values of n_s.

Effect of Different Scalar Spectral Index Values:

- n_s = 0.96: The power spectrum is more tilted (steepest curve), indicating a greater difference between large-scale and small-scale power. This corresponds to a universe where the largest scales dominate the structure formation.
- n_s = 0.965: This is closer to the current observational value from Cosmic Microwave Background (CMB) data. The curve is still tilted but less steep than for n_s = 0.96, indicating slightly more uniform power across scales.
- n_s = 0.97: The curve flattens further, indicating less difference between power on large scales and small scales. This results in a spectrum that is closer to scale-invariance (where power is distributed evenly across different scales).

The amplitude A_s(set to a constant value in the plot) determines the overall scale of the power spectrum. Changes in $n_s$only affect the tilt of the curve, while $A_s$controls the absolute level of the perturbations.

Hence our plot for the power spectrum provides insights into the initial conditions of the universe set by inflation.

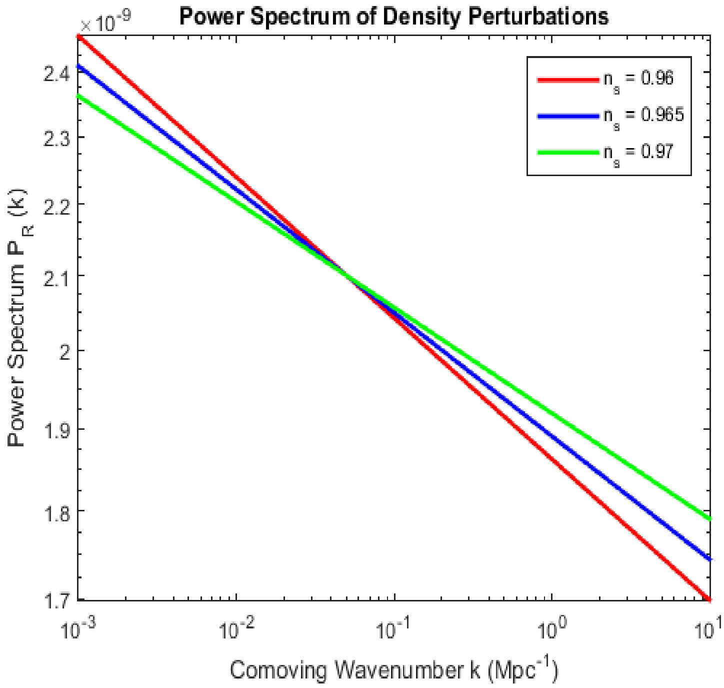

Fig. v) Power Spectrum P_R(k) vs. Comoving Wavenumber k(Mpc^{-1}) Curve

Black hole stable The rate of mass loss due to Hawking radiation ($\frac{dM}{dt}$) as a function of black hole mass (M) :

The graph shows an inverse relationship between the black hole mass.M and the rate of mass loss $\frac{dM}{dt}$. As the mass increases, the rate of mass loss decreases significantly. This trend is due to the M^2 term in the denominator of the mass loss equation, which implies that larger black holes radiate energy at a much slower rate compared to smaller ones. For black holes with smaller masses (e.g., stellar-mass black holes), the rate of mass loss is relatively high. The graph sharply declines as mass decreases, indicating that small black holes (e.g., those with less than a few solar masses) would evaporate quickly compared to their more massive counterparts.

Supermassive black holes, which are located in the centers of galaxies, experience a mass loss rate that approaches zero. This implies that these black holes are stable over astronomical timescales and can persist for billions of years without losing significant amounts of mass.

Additionally, the data from the graph emphasizes the importance of quantum effects in the physics of black holes since Hawking radiation is a quantum phenomenon. This highlights the ongoing efforts in theoretical physics to unify general relativity with quantum mechanics.

In summary, the plotted graph clearly illustrates the significant relationship between black hole mass and the rate of mass loss due to Hawking radiation. It offers important insights into black hole stability, lifetime, and the dynamic nature of these intriguing objects in the cosmos.

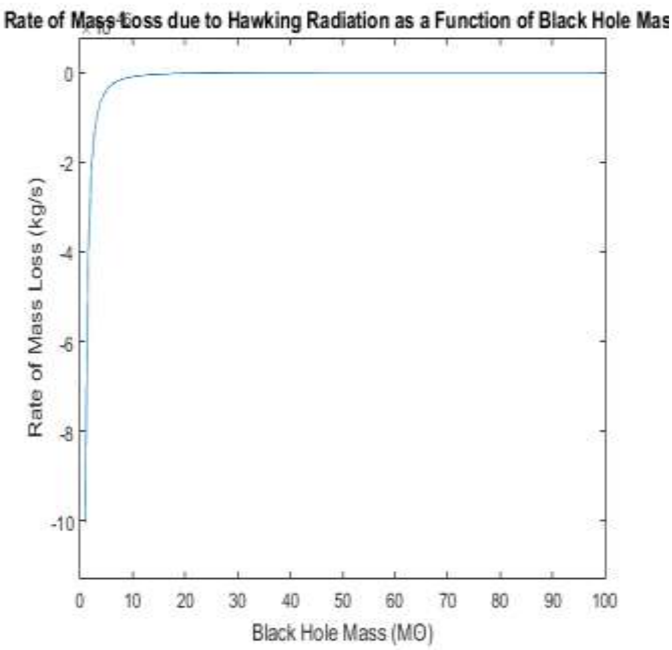

Fig. vi) Rate of mass loss due to Hawking Radiation (kg/s) vs. Black Hole Mass (MΘ) Curve

The Lifetime (τ)of a Black Hole due to Hawking radiation as a function of the mass of a black hole (M) :

The graph demonstrates a cubic relationship between black hole mass and lifetime, as indicated by the derived equation:

$$\tau \propto M^3$$

As a black hole's mass increases, its lifetime also increases significantly, specifically by the cube of the mass. For instance, a black hole with 10 solar masses has a lifetime that is 1,000 times longer than that of a black hole with 1 solar mass.

The y-axis of the graph is on a logarithmic scale, which helps illustrate the wide range of lifetimes more clearly. This scale makes it easier to compare the long lifetimes of massive black holes. In contrast,

smaller black holes (those with a mass below a few solar masses) exhibit rapidly increasing lifetimes, as shown by a steep curve that indicates they lose mass quickly due to Hawking radiation.
In the case of supermassive black holes—ranging from hundreds of thousands to billions of solar masses—the curve flattens. This means that while they have very long lifetimes, adding more mass contributes less to increasing their lifetime.
Overall, the graph demonstrates that smaller black holes evaporate much faster than larger ones, suggesting that primordial black holes from the early universe have largely evaporated, leading to a scarcity of small black holes today. The plotted curve offers key insights into the relationship between black hole mass and lifetime, underscoring their significance in understanding black hole dynamics and their role in the universe.

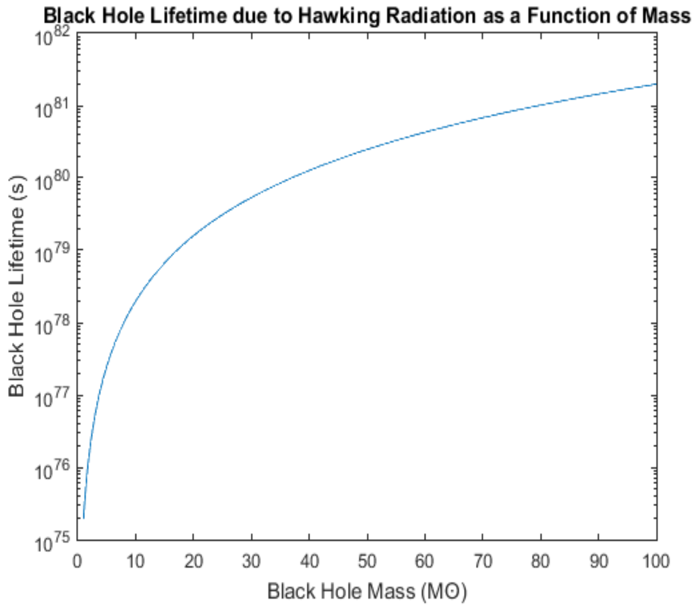

Fig. vii) Black Hole Lifetime (s) due to Hawking Radiation vs. Black Hole Mass(MΘ) Curve

4. CONCLUSION

Our research explored how black holes formed in the early universe and why this study is essential. Understanding black holes is crucial for comprehending cosmic evolution and galaxy formation. We applied theoretical concepts such as the Friedman equations(Friedmann, 1924), Jeans instability(Jeans, 1902), and Hawking radiation(Hawking, 1975) to investigate the conditions for black hole formation and growth during the universe's infancy.
These concepts shed light on the expansion of the universe and the gravitational collapse of matter, emphasizing that black holes, particularly supermassive ones, significantly influence galaxy evolution. This study highlights that black holes are dynamic entities that shape the universe, providing vital insights into early formation processes and ongoing cosmic evolution. The insights gained from this study deepen our understanding of early black hole formation and offer essential clues about the ongoing processes that govern cosmic evolution and structure formation throughout cosmic time.

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