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Growth Curve of Leaf of *Ficus benghalensis L.* and Mathematical Modeling of Spiral Phyllotaxy.

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Abstract: Nature follows mathematical rules in Phyllotaxy and in-leaf morphogenesis. In a case study, Post abscission due to summer vacation, the small banyan tree Ficus benghalensis L. (Banyan) on the pot was fallen leaves. Therefore it was given water and two day after the rain started and recovered regeneration of leaves, the growth is measured in vivo, and calculated increase of length, width, and leaf surface area. and a Growth curve graph plot was made. The classical mathematical model of the phyllotaxy of Banyan is discussed in the view of parastichy Fibonacci number , Lucas numbers and golden ratio theories.
Key Words Ficus benghalensis L. Leaf Growth Measurement , leaf surface area , Leaf Growth curve. Phyllotaxy.

INTRODUCTION:

Mathematics in Forest ecosystem is studied such that, frequency, Density, abundance, Basal area and important value index of the forest Trees. Phyllotaxy and aestivations are also mathematically well described. Graph Plot method to measure the leaf surface area is Simple method but growth formulae are described in dry weight .because If, dry no biomass.[1,2,3,4,5].

Lyman Briggs and Homer Le Roy Shantz (1912) proposed the wilting coefficient, which is defined as the percentage water content of a soil when the plants growing in that soil are first reduced to a wilted condition from which they cannot recover in approximately saturated atmosphere without the addition of water to the soil. [6, 7]

Frank Veihmeyer and Arthur Hendrickson from University of California-Davis found that it is a constant (characteristic) of the soil and is independent of environmental conditions. Lorenzo A. Richards proposed it is taken as the soil water content when the soil is under a pressure of −15 bar[8, 9]

Wilting diminishes the plant's ability to transpire and grow. Permanent wilting leads to plant death. Symptoms of wilting and blights resemble one another. The plants may recover during the night when evaporation is reduced as the stomata closes. In woody plants, reduced water availability leads to cavitations of the xylem. Cavitations is a phenomenon in which the static pressure of a liquid reduces to below the liquid's vapour pressure, leading to the formation of small vapor-filled cavities in the liquid**.** Wilting occurs in plants such as balsam and holy basil. Wilting is an effect of the plant growth-inhibiting hormone, abscisic acid. [10, 11, 12, 13]

Botanists of Bio-Industrial Engineering, National Taiwan University, Taiwan, R.O.C studied automation recovery in natural plant body system with the favorable change in the Environmental factor in some plants [14] Soil moisture, high Hygroscopic air and reduced temperature helps the plant to recover the damage due to draught and for regeneration .[15]

Finally, the leaf grows and its cells undergo cell-fate determination and differentiation during secondary morphogenesis **.** During its development, the different layers of the leaf, its vasculature and specialized epidermal cells, such as trichomes and stomata guard cells, undergo differentiation.[16,17]

MATERAL AND METHOD:

On Dated 27 June2023, took photograph in Figure1 represents that, at my Lab.- window, grown Banyan plant is tuff to imagine without its leaf. Same in other Pot the small Boe Plant too got dried due to summer vacation and no watering to the plant on small pots. Therefore pots were given full water. Resulting to Recovering of Banyan plant and Boe pant was not recovered gone to permanent wilting. The Banyan Plant *(Ficus benghalensis L)* was fully recovered and leaves were regenerated to the full size until 13 July2023 as depicted in Figure2.The length width was measured with meter tape as depicted in Figure3,Figure4 and Figure 5. The area and perimeters of the leaf is calculated by graph plot method.[18] The fresh Leaf of Banyan of Same size (Length and width) is weighted and dried in shade up to six days below Fan 'on' and the dried leaf was again weighted and the moisture content and dry weight is noted on 21 August 2023.. Figure 12 represents adaxial surface (stem side) of the leaf, Figure 13 represents abaxial surface of leaf opposite side where mid rib highly ridged. Dry leaf is weighted on an electronic balance and dry weight was found 2.32 gram.(19)

Table 1 Measurement of growth rate of Banyan Leaf:

Sr.	Time	Date	Length of leaves	Width of the leaf	Area of leaf	Used Time	Growth Rate= Area/ Day
1	11.00 a.m.	27/6/2023	00	00	00	00	00

2	11.00 a.m	01/07/2023	6.5 cm	3.2 Cm	15 Cm^2	4 days	3.75 Cm^2/day
3	11.00 Am	05/07/2023	10,0 Cm	5.6 Cm	44 Cm^2	8 days	5.5 Cm^2/day
4	11.00 a.m.	09/07/2023	16.00 Cm	8. 4 Cm	80 Cm^2	12 das	6.66 Cm^2/day
5	11.00 a. m.	13/7/2023	18.6 Cm	10.0 cm	110 Cm^2	16 days	6.87 Cm^2/day
6	11.00 a. m.	18/07/2023	18.6 Cm	10.0 cm	110 Cm^2	21 days	5.23 Cm^2/day

Leaf surface area by graph plot method Leaf surface area is obtained by graph plot method. It was placed the leaf to be measured on graph sheet that has 1 cm square grid lines. Outline the perimeter of the leaf on the graph paper. Counted the number of square blocks are enclosed inside the leaf outline. Finally, the leaf surface area is calculated using equation,, such that, Leaf Area = NGS × OGA**.** Where, NGS is the number of grid squares inside the leaf outline and OGA is one grid area. The precision of this technique is dependent on, how the areas covered by half grid square and quarter grid squares are accurately interpreted. Figure 12 represents the measurement of leaf surface area of *Ficus Benghalensis L i.e.* Banyan name given by Linnaeus [20]

OBSERVATION AND RESULT; *Ficus Benghalensis* plant's observation started on Dated 27 June2023,and Final observation was on 13 July2023, thus total days during leaf generation and growth =18days = 432 hours.
First reading on 27 June as in Figure 1 the leaf development is not started , therefore first reading of leaf size =00mm.and G_o =0.
Final reading on 13 July2023, of Banyan leaf length with petiole = 18.6Cm. = 186mm.
And width on Final reading =10 cm=100mm. ^ Area of leaf by graph plot method =110 Cm^2.
Weight of the fresh Banyan leaf =6.25 grams. ^ Dry weight =2.32 gram.
Moisture evaporated water = 6.25- 2.32 = 3.93 gram.
% water in fresh Banyan leaf = (3.93 / 6.25) x100 = 62.88% ……Eq1

- From final measure(on 21st August 2023) of Dry weight of Banyan leaf :
  =2.32 gram. 232 x 10^4µgram /18 days = 24 x 10^4µgram / days = 10,000 µgram/ Hour.
  = > 10,000/60=166.66 µgram Per minute and finally 2.77 µgram per second…..Eq.2.

GROWTH CURVE: Growth is permanent and irreversible increase in any dimension of an organism. The growing oint is called Meristem. There are three main actions in the meristem: Cell division, cell enlargement and cell differentiation. The growth of a plant or a plant organ is done by cell division and cell enlargement up to same size of mother cell, before they divide again. Thus the growth of a plant or plant organ is essentially an increase in the size due to an enlargement of cells and this enlargement must be irreversible due to normal increase in hydrostatic pressure caused by turgidity.
Thus, Growth=$W_1=W_0^{r.t}$ = Log_e (W_1/W_0) = r t
= > 2.3026 log (W_1/W_0) = r t
= > r = 2.3026 log(W_1/W_0) / t……Eq.3
Where, W_0= initial dr7 weight ,
W_1= Final dry weight.
e = Base of natural logarithm
r = Rate of increase in dry weight/Unit time, and
t= Time interval .Since here G_0=0 µgram. Therefore Eq 3 is not applicable. Results are already satisfactory by Eq1,and Eq.2. Since the growth of Leaf surface area is in the proportional of increase of its dry weight, Therefore, the graph plot is made between Time and leaf surface area. Figure 7 represent the Growth curve of the leaf of *Ficus benghalensis.* In which, *T*ime is in X axis, with a scale *i.e.* One Big Square = 48 Hours.(2days) and Growth of the leaf area is put in Y axis, where, one Big Square = Area 10Cm^2. The data for Time and leaf area is taken from Table1, and Growth curve plot is depcted in Figure 7.

LEAF: Figure 6 represents that banyan leaf is petiolated (Short and thick), lamina is oval in outline, it's middle part is widest, and the end is slightly narrow and round and smooth margin , this shape of lamina is called Elliptical leaf. Is has Unicostate Reticulate venation . when the got dry , its ovate form changes into triangle. When it is measured dry weight. Fugure 15 represents tthat There is an Acute triangle ADC at tip side plus at adjacent to petiole an obtuse triangle.ABC: In △ ACD, ∠ADC =50* and ∠DAC= ∠ACD=65* In ⌓ ABC, ∠ABC=120* AND ∠BAC= ∠ACB = 30*. The angle between Veins and mid rib are mostly 60° but at the base 90° inside obtuse triangle..

PHYLLOTAXY; The l arrangement of leaves, called phyllotaxy, It obeys a number of subtle mathematical relationships. The classical mathematical theory of phyllotaxy makes it possible to model all the arrangements from the angular and radial positions. The objective of this phyllotaxy is to avoid shading one another, so that the leaves may get the maximum sunlight for photo physiology . The mathematical models of phyllotaxy have been developed based only on a few parameters.[21,22]

MATHEMATICAL MODEL: There is a definite law according to which the leaves are arranged on the stem o a particular plant species. One could easily verify that :-
1..The angular distance (angular divergence) between two consecutive leaves is constant.
2.When Looked from the top, all the leaves are found in a fixed number of vertical rows called Orthstichies.
3.The Orthstichies are evenly dispersed on a circle and the angle between adjacent Orthstichies being cconstant.
4.Although, leaves look scattered, a close examination shows that leaves are evenly dispersed on all sides of the stem.

If we imagine a line touching the stem by leaves, there is a spiral line, hence it is named spiral phyllotaxy. Genetic Spiral could easily be projected on a flat surface to form a flat spiral and the position of the leaves were marked on the flate spiral, as shown in figure 9. The angle subtended at the centre by two consecutive leaves is the **angular divergence**. It could be denoted by finding out a leaf which is exactly above a particular leaf, then by counting the number of complete circle covered by the genetic spiral, and the number of leaves beginning from the first to the one just before the last. That number of leaves results the number of Orthstichies.(vertical rows). Thus, The angular divergence= number of circle / Number of leaves or Orthstichies of a circle (360^0). Figure 8 represents that the sixth leaf is stand over the first leaf. The first leaf and genetic spiral complete two circles to come over the sixth leaf. The seventh leaf stands over the third, ninth over the fourth, the tenth over the fifth, and eleventh over the sixth and first leaf. Tthus , there ar 5 Orthstichies *i.e.* leaves are arranged in five rows, and because two turns of geneti spiral nvolve five leaves, the latter are seen to be placed the the two-fifth the distance of the circle. Phyllotaxy is therefore 2/5 o five ranked or **pentastichous phyllotaxy.** The angular divergence is 2/5 of 360^0 *i.e.* $720/5= 144^0$. Golden angle is calculated by divide perimeter in golden ratio

$\Rightarrow a + b /a = a/ b \rightarrow a b = \theta > \alpha = 360^0 b /(a + b) = 360^0 / \alpha^2 \approx 137.5^0$

In spiral phyllotax successive leaves grow at approximately Golden angle $\theta= 137.5^0$ Here, in The pentastichous phyllotaxy, deference of the divergence angle is $144-137.5= 6.5^0$ In Fibonacci numbers, Lower limit =0 , $F_1 = 1$ and $F_2 = 1$. `.` Fn = Fn-1 + Fn-2. .`. Sequence of Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, … Lucas numbers $L_1 = 1$ $L_2 = 3$ Ln = Ln-1 + Ln-2 Sequence of Lucas numbers 1, 3, 4, 7, 11, 18, 29, 47, 76……The numbers Statistics for Phyllotaxis *of Ficus Benghalensis* is 95% Fibonacci numbers 4% Lucas numbers 1% not properly formed. [23]

Figure1 Wilted Plants

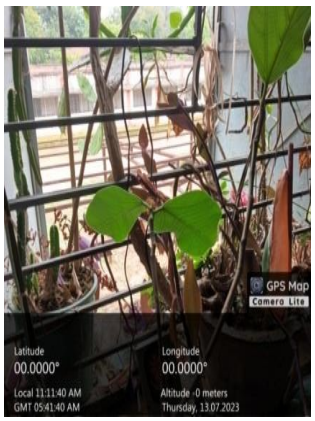

Figure2Recovered Plants

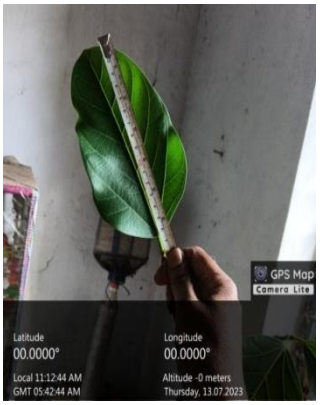

Figure3 Length of Banyan leaf

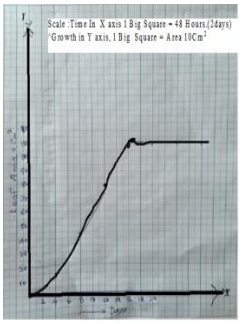

Figure 4 Full Length Figure 5 Width of Banyan Figure 6. Leaf area Figure 7 Growth curve


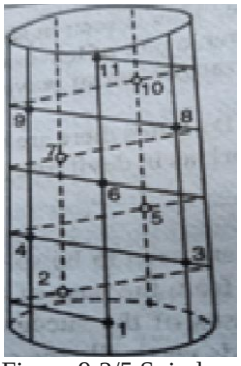
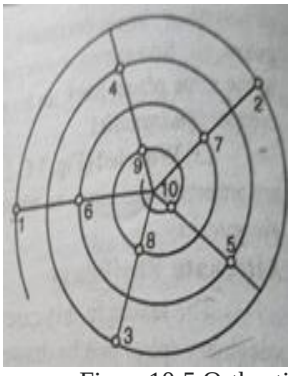
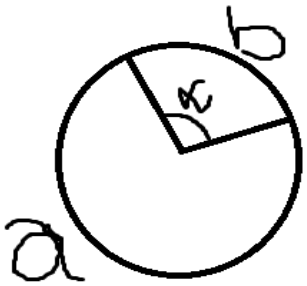

Figure8 Spiral Phyllotaxy Figure 9:2/5 Spiral Figure10:5 Orthostichies Figure 11 Golden angle



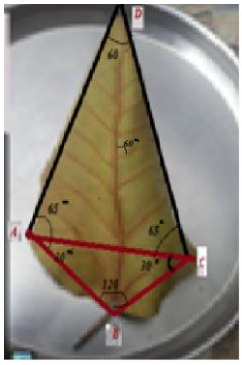

Figure 12 Adaxial Figure13abaxial Figure14Dry weight of leaf. Fig15 Trigonometry of dry leaf

DISCUSSION:

In the Spiral phyllotaxy, a new primordia start growing at the site where plant hormone auxin is depleted. Meristem Auxin hormones are released by growing primordia. New primordium wants to be as far apart as possible from the existing primordia. Mechanical analog with magnetic repelling particles with a force proportional to 'd' (where 'd' is their distance). These dipoles are adverted by a radial gradient of the magnetic field . their velocity V(r) being limited by the viscous friction of the oil. In order to model the apex, the dish has a small truncated one at its center. As the plant is growing it is gradually reducing the time delay between formations of new primordia. The spiral patterns then go sequentially through all the Fibonacci parastichies. Occasional excursions to the neighbor local minima produce Lucas parastichy numbers.

At Presidency College Colcutta.09 WB India. Famous Indian Physician and Botanist Sir J C Bose , measured growth rate of coleoptiles of *Zea maise* to be 35 meu per second at optimum condition. Here growth was assumed length of coleoptiles (plumule or stem primordial) Tactic movement of plant is permanent growth on any direction of favorable stimulation like the light. Phototropism is a plant response of the light stimulus. In his book, Life Movements in Plants He measured highest Rate of Flowers of *Datura* is about 75meu Per second .

He also measured the force used for angular momentum of growth of tendrils of Pea and *Cucurbita*. He invented **crescograph** to measure plants growth in length by optical micrometer .He experimented on

The electrical, gravitational and thermal stimulations on plant growth response along with the study on Positive Geo tropism is growth of root tip. [28, 29]

American Scientist, F W Went experimented to study the effect of plant hormone Auxin on the growth, and tropisms in decapitate Colioptile.of *Avena Sativa* (Oat). It was the phototropic growth in curvature in the length of *Avena sativa* (oat)

There are environmental factors affecting to the growth of plants and plant parts . E.g. Light : Since , experimental plant of Banyan is on the ever open window getting diffused light only. Hence its leaf is dark green. And less (only 3) in number. So that , the other leaf of banyan of same size is weighted to found 6.25 grams. It is dried and dry weight is taken for calculation of growth rate.[30]

It has been assumed that growth is dependent on chemical transformation of reverse substance into new living protoplasm. The new protoplast as soon as it is formed begins to participate in the mechanism of transformation by apparently catalyzing it's own synthesis b autocatalytic reaction . by this principal the theoretical growth curve obtained is Sigmoid. (S shaed) as depicted in figure7.

FURTHER SCOPE OF THE STUDY:

Nature, mathematics, and time are an infinite and follow the mathematical axiom of union. A tree has a main stem and many branches and further branching and twigs, There are many leaves on a twigs and branches along with the flowers to transform into fruits, containing inside seeds. Generally main branch arise in 45° from stem. Because Tan 45°=1. And this is axiom of union in nature. This helps the main stem to balance the weight of the main branch which goes further repeated branching. There is a definite angle on the origin of rosette leaf (phyllotaxy) is such that the sun light fall on each leaves. Some sun loving trees or its branches dry in shade either shade of itself or of other. And definition angle of flower to be in more contact of air for wind pollination, and seed dehiscence is found in their Inflorescence to give rise from stem twig with peduncle.

If there is microtome with Meu meter adjusting section could be and cells diameter be measured by micrometer and the number of cells in the leaf could be counted . A mathematical relation between the various parts of leaf anatomy, *i.e.* epidermis, hypodermis, palisade and vascular bundles could be found and further more in a Vascular Bundle the mathematical relation of xylem phloem and parenchyma in number, volume and dry weight. Thus 3x3 square Matrix could be constructed. The Matrix determination value and calculation of co factor could in an interesting mathematics.

CONCLUSION:

1. Percent moisture in fresh Banyan leaf is 62.88% ➖
2. Growth Rate of the leaf (in dry weight is)2 24 x 10^4µgram / days = 10,000 µgram/ Hour.
   = 2.77 µgram per second. ➖
3. Growth curve of the leaf is opposite L shaped since leaf growth starts as ridges and got stable up to long time. Therefore in standard Sigmoid "S" Growth curve upper and lower curves of S is absent being opposite 'L' shape.
4. Mathematical modeling of spiral phyllotaxy of *Ficus benghalensis L* . is pentastiechy or 2/5 phyllotaxy . *i.e.* there are leaves arranged spirally , where 5 vertical rows of leaves arise. The sixth leaf is upon the first leaf round two circles. 360x2 /5=(72° x2) =144^0angle between adjacent leaves.
5. After full dry, an Ovate leaf changes to triangular by folding adaxial inside and midrib highlighted abaxial outside.

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