IJRAR.ORG E-ISSN: 2348-1269, P-ISSN: 2349-5138

INTERNATIONAL JOURNAL OF RESEARCH AND ANALYTICAL REVIEWS (IJRAR) | IJRAR.ORG

An International Open Access, Peer-reviewed, Refereed Journal

JOURNEY OF FIXED POINT THEORY ON DIGITAL PATHWAY: A BRIEF REVIEW

Sangeeta Pandey[1], Deepak Kumar Tiwari[2]

[1]Department of Mathematics, Rajeev Gandhi Govt. P.G. College, Ambikapur, Surguja, Chhattisgarh, India

[2]Department of Mathematics, Shri Sai Baba Aadarsh Mahavidyalaya, Ambikapur, Surguja, Chhattisgarh, India

Abstract:

Information Technology plays a vital role in every step of human activity in 21st century. In this era we can not imagine our life without technology because the virtual communication take place over traditional communication by using various ways like video conferencing, telephoning, E-mailing, Image processing etc. On the other hand fixed point theory is very traditional topic of pure part of Mathematics which plays a vital role in many disciplinary of Mathematics such as Topology, Physics, Chemistry, Biology, Computer Science etc. In second phase of 20th century researchers tried to provide a digital version of this theory.

Keywords: Fixed Point, Digital Space, Information Technology, Mapping.

Introduction:

The theory of fixed points is now a promising area in mathematics especially in Non-linear Functional Analysis because it has wide applicability in various fields of pure and applied Mathematics as well as in other fields like Physical Science, Life Science and Economics.

The field of the fixed point theory is expanding its domain, thereby leading to the emergence of a plethora of techniques and ideas. The fixed point theory is one of the very effective and fruitful tool in mathematics which has huge applications within as well as outside the mathematics. It is a beautiful mixture of Analysis, Geometry and Topology.

Fixed point theory is influential field of specialization and has grown into a full branch of mathematics with in the span of more than hundred years out of multitudes problems occurring in diversified fields. It is very difficult to imagine its applications in many fields.

Now a days human world has became completely digital. In every step of life human uses technology in every sector such as Medical, Education, Defence, etc. From these sectors Image processing is a promising area in which fixed point theory is more applicable. Many researchers try to explain Image processing with fixed point theory. In this way we provided a brief review on digital version of this theory in this paper.

Survey of Literatures

S.No.	Year	Title of Paper	Authors	Journal	Volume	Page Number
1	1961	An extension of Banach contraction principle.	M. Eldestein	Proceeding American Mathematical Society	12(1)	7-10
2	1979	Digital Topology	A. Rosenfeld	American Mathematical Society	86	621-630
3	1986	Continuous functions on digital pictures	A. Rosenfeld	Pattern Recognition Letter	4	177-184
4	1994	Digitally continuous functions	L. Boxer	Pattern Recognition Letter	4	833-839
5	2001	A digital version of the Kakutani fixed point theorem for convex-valued multifunctions	M.B. Smyth and R. Tsaur	Electronic notes in theoretical computer science	40	15
6	2013	Lefechetz fixed point theorem for digital images	O.Ege, I. Karaca	Fixed point theory and applications, Springer Open Access journal.		
7	2015	Banach fixed point theorem for digital images	O.Ege, I. Karaca	Journal of nonlinear Science applications	8	237-245
8	2015	Fixed point theorems for digital images,		Honam Mathematical Journal	37(4)	595-608
9	2016	Common fixed point theorems in digital metric spaces.	A. Rani and K. Jyoti	International Journal of Scientific & Engineering Research	7(12)	1704-1716
10	2016	Banach fixed point theorem from the viewpoint of digital topology	S.E. Han	Journal non linear Science Applications	9	895-905
11	2017	Digital expansions endowed with fixed point theory	K. Jyoti and A. Rani	Turkish Journal of Analysis and Number Theory	5(5)	146-152
12	2017	Banach and Eldestein fixed point theorem for digital images	A. Hossain, R. Ferdausi, S. Mondal and H. Rashid,	Journal of Mathematical Science and Applications	5	36-39
13	2017	Fixed point theorems for digital contractive type mappings in digital	K. Sridevi, D.M.K. Kiran, and M.V.R. Kameswari,	International Journal of Mathematics Trends and Technology	48(3)	159-167

		metric spaces				
14	2018	Fixed point Theorems with digital Contractions image processing	L.N. Mishra, K. Jyoti, A. Rani and Vandana	Non linear Science letter	9(2)	104-115
15	2018	Common fixed point theorem for intimate mappings in digital metric spaces	D. Jain	International Journal of Mathematics Trends and Technology	56(2)	91-94
16	2018	Fixed point theorems in digital images and applications to fractal image compression	U.P. Dolhare and V.V. Nalawade	Asian Journal of Mathematics and Computer Research	25(1)	18-37
17	2019	A contraction mapping method in digital image processing	A. Mishra, P.K. Tripathi, A.K. Agrawal and D.R. Joshi	International Journal of Recent Technology and Engineering	8(4S5)	
18	2019	Application of fixed point theorems for digital contractive type mappings in digital metric spaces	N. Gupta, A. Singh and G. Modi	International Journal of Scientific Research in Mathematical and Statistical Sciences	6(1)	323-327
19	2021	Common fixed point theorems in digital metric space using implicit relation	A. Suganthi	Advances and Application in Mathematical Sciences	20(10)	2351-2369
20	2021	On digital metric space satisfying certain rational inequalities	K. Shukla	An International Journal of Application and Applied mathematics	16(1)	223-236
21	2022	Fixed point theory in digital topology	R. Kalaiarasi and R. Jain	International Journal Nonlinear Anal. Appl.	13	157-163

Methodology:

For researchers literature review is an important part of their research. In this paper, we are presented some reviews of published research papers related to digital version of fixed point theory. Our research work is totally theoretical and we apply survey method for this work.

Description of Survey:

Eldestein calculated and proved some common fixed point theorems in compact and Hausdroff spaces for couple of mappings with some new condition of contraction. This was an extension of Banach contraction principle (Eldestein, 1961).

Rosenfeld blended a new theory known as "Digital Topology". In his work he analyzed digital pictures which is frequently involves segmenting it into pieces and evaluating different possessions of digital images. This type of property is referred to as Digital Topology (Rosenfeld, 1979). Again, he discussed on continuous functions on digital photographs. In this work he recycled the meaning "lattice point" to describe and prove the continuity of functions. He directly described that the way of a continuous function commencing a finite slab of lattice points into itself has fixed point property (Rosenfeld, 1986). This work was extended by Boxer with investigated the digital form of homeomorphism, retraction and homotopies (Boxer, 1994).

Smyth and Tsaur discussed on fixed point theorems. they enlightened the notion of power graph and power complexes. They also cleared that the concept of weak convexity and having fixed point property which reflected as the digital version of Kakutani fixed point theorem (Smyth & Tsaur, 2001).

Ege and Karaca, discussed on FPP of digital images and proved Lefchetz fixed point theorems (LFPT) for digital images and in 2014, they introduced some applications of LFPT to digital images. The degree of antipodal map was also assumed for sphere like digital images (Ege & Karaca, 2013 & 2014). In 2015, they introduced the concept of digital metric space and established Banach fixed point theorem for digital images (Ege & Karaca, 2015). Han established some fixed point theorems for digital images using BCP and enlightened the BCP for digitally metric space and also stretches a significant outcome that digital metric space is complete. From the point of view of digital images he established Banach fixed point theorems (Han, 2015 &2016).

Rani and Jyoti introduced commutative, weakly commutative, compatible weakly compatible mappings by setting of digital metric space and proved some common fixed point theorems by using properties of these mappings in digital metric space also described the image processing (Rani & Jyoti, 2016).

Devi and Dmk discussed on fixed point. They introduced contraction and contractive mappings in digital metric space. They proved a lemma and by using this established the existence and uniqueness of fixed points in digital metric space (Devi et.al, 2017).

Hossain et.al considered the properties of digital topology and generalized Eldestein fixed point theorem for chainable metric spaces which was referred as Banach and Eldestein fixed point theorems for digital images (Hossain et.al 2017).

Jyoti et.al discussed on zooming in and zooming out of the digital photographs and assigned expansive mappings as digital expansions for established some fixed point theorems in complete digital metric spaces (Jyoti et.al, 2017)

Jain discussed on fixed point in digital metric space and proved some common fixed point theorems using some intimate mappings in this space (Jain, 2018). In the same year Mishra et.al discussed on fixed point

and used the concept of digital contractions to established some new fixed point results in digital image processing (Mishra et.al, 2018).

Dolhare and Nalawade discussed on fixed point and ideas of digital images and provided a digital version of some important generalizations of BCP. They extended the BCP to digital image by using a non decreasing function. Also they established another fixed point theorem by using weakly uniformly strict contraction. They also described an application of these theorems to fractal image compression and enhance the perceptual quality of transformed digital images (Dolhare & Nalawade, 2018).

Gupta et.al introduced θ type contraction and contractive mappings on digital metric space and by using these proved the existence and uniqueness property of fixed points in digital metric space. These result shows the applications of FPT in digital metric space (Gupta et.al, 2019).

Mishra et.al established the existence and uniqueness of fixed points and enlightened how to use contraction mapping method in digital image processing. They concluded that In digital image processing, fixed point theorems may be used in compressing the size and improving the image quality by using contraction principle. Since after processing the quality of compressed image is not obtained so good every time even, it could not carry the same information as the source of image. Thus by this result they reinforce the idea of digital topology especially, in image processing to compress the image with least redundancy (Mishra et.al, 2019).

Shukla discussed on fixed point theory on digital metric space with certain rational inequalities. The work addressed in this research work by extending some existing theorems like Kannan, Chatterjea and Reich contraction theorems by the setting of digital metric space (Shukla, 2021).

Suganthi introduced some new common fixed point theorems for a class of contractive type mappings in digital metric space by using implicit relation. In this work a generalized version of BCP in digital metric space was proved along with an application image processing in digital metric space (Suganthi, 2021).

Kalaiarasi and Jain discussed on fixed point theory in digital topology. They mentioned the basic concept of digital images and reviewed some research works on exploring image processing in digital spaces using fixed point theorems. Also they proved some fixed point theorems on digital metric spaces by replacing the conditions in existing theorems with a suitable condition (Kalaiarasi & Jain, 2022).

Conclusion:

From the above review we concluded that digital version of fixed point theory and metric space is an interesting domain of research. Due to advancement of technologies, to point out and correlate the fixed point theory with digital metric space is a challenging issue for researchers. In current scenario some research were done in digital metric space with establishment of fixed point for contraction and expansive mappings but there are needed more research in this field to develop Mathematical concepts in Information Technologies.

References:

[1]. Boxer, L. (1994), "Digitally continuous functions." Pattern Recognition Letter, Vol. 4, pp. 833-839.

[2]. Dolhare, U.P. and Nalawade, V.V. (2018), "Fixed point theorems in digital images and applications to fractal image compression." Asian Journal of Mathematics and Computer Research; Vol. 25(1), pp. 18-37.

[3]. Ege, O.,Karaca, I.(2013), "Lefechetz fixed point theorem for digital images." Fixed point theory and applications, Springer Open Access journal.

[4]. Ege, O., Karaca, I. (2015), "Banach fixed point theorem for digital images." Journal of nonlinear Science applications, Vol. 8, pp. 237-245.

[5]. Eldestein, M. (1961), "An extension of Banach contraction principle." Proceeding American Mathematical Society; Vol. 12(1), pp. 7-10.

[6]. Gupta, N., Singh, A. and Modi, G. (2019), "Application of fixed point theorems for digital contractive type mappings in digital metric spaces." Interntational Jornal of Scientific Research in Mathematical and Statistical Sciences, E-ISSN: 2348-4519; Vol. 6, Issue-1, pp. 323-327.

[7]. Han, S.E. (2015), "Fixed point theorems for digital images." Honam Mathematical Journal, Vol. 37(4), pp. 595-608.

[8]. Han, S.E. (2016), " Banach fixed point theorem from the viewpoint of digital topology." Journal non linear Science Applications, Vol. 9, pp. 895-905.

[9]. Hossain, A., Ferdausi, R., Mondal, S., Rashid, H. (2017), "Banach and Eldestein fixed point theorem for digital images." Journal of Mathematical Science and Applications, Vol. 5, pp. 36-39.

[10]. Jain, D. (2018), " Common fixed point theorem for intimate mappings in digital metric spaces." International Journal of Mathematics Trends and Technology; Vol. 56 (2), pp. 91-94.

[11]. Jyoti, K. and Rani, A. (2017), "Digital expansions endowed with fixed point theory." Turkish Journal of Analysis and Number Theory, Vol. 5(5), PP 146-152.

[12]. Kalaiarasi, R. And Jain, R. (2022), "Fixed point theory in digital topology." International Journal Nonlinear Anal. Appl., Vol. 13, pp. 157-163.

[13]. Mishra, A., Tripathi, P.K., Agrawal, A.K. and Joshi, D.R. (2019), "A contraction mapping method in digital image processing." International Journal of Recent Technology and Engineering, ISSN: 2277-3878, Vol. 8, Issue-4S5

[14]. Mishra, L.N., Jyoti, K., Rani, A. and Vandana (2018), " Fixed point Theorems with digital Contractions image processing." Non linear Science letter, Vol. 9(2), pp. 104-115.

[15]. Rani, A. and Jyoti, K. (2016), "Common fixed point theorems in digital metric spaces." International Journal of Scientific & Engineering Research, ISSN: 2229-5518, Vol. 7, Issue 12, PP 1704-1716.

[16].]. Rosenfeld, A. (1979), "Digital Topology." American Mathematical Society; Vol. 86, pp. 621-630.

[17. Rosenfeld, A. (1986), "Continuous functions on digital pictures." Pattern Recognition Letter; Vol. 4, pp. 177-184.

[18]. Shukla, K. (2021), "On digital metric space satisfying certain rational inequalities." An International Journal of Application and Applied mathematics, Vol. 16, Issue-1, pp. 223-236.

[19]. Smyth, M.B. and Tsaur, R. (2001), "A digital version of the Kakutani fixed point theoremfor convex-valued multifunctions." Electronic notes in theoretical computer science, Vol. 40, page 15.

[20]. Sridevi, K., Kiran, D.M.K., and Kameswari, M.V.R. (2017), "Fixed point theorems for digital contractive type mappings in digital metric spaces." International Journal of Mathematics Trends and Technology; Vol. 48 (3), pp. 159-167.

[21]. Suganthi, A. (2021), "Common fixed point theorems in digital metric space using implicit relation." Advances and Application in Mathematical Sciences; Vol. 20 Issue 10; PP 2351-2369.